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Accelerated simulation of hierarchical military operations with tabulation technique

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Recently, a challenge in defence modelling and simulation is that simulating a satisfactory number of scenarios often requires an infeasible runtime. This paper resolves this challenge by utilizing a tabulation technique that encourages reuses of the previous simulation results in hierarchical models. For example, a mission-level model may contain a number of similar engagement scenarios that must be executed multiple times by an engagement-level model. Therefore, we collapse the multiple similar executions into a single simulation run while verifying the statistical stability in the output distribution. This reuse is supported by adapting the tabulation technique to hierarchical models with the extension of an interpolation in matching the lower abstractions. An application in the naval air defence domain shows that the simulation is speeded up a maximum of seven times, while producing statistically identical simulation results with few increments in the variance.

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1. Introduction

Military commanders spend significant amount of time in preparing for battles that take very little time to play out in the real world. This preparatory period includes training units, maintaining equipment, and planning the course of action (COA). Each of these preparations involves the repetition of exercises assumed in battles, which ensures that units, the equipment, and the plans are ready to handle any surprises in the real battles. Therefore, anticipating unexpected events is a critical task in preparing for battles, and defence modelling and simulation (DM&S) plays an important role in identifying such events by repeating many battle simulations, known as *war games* (Dunnigan, 2005)

Turning our focus to the development of COA in the military decision-making process (Department of the Army Headquarters, 1997) (see Figure 1), we see that the military commanders and staff members perform the COA analysis. The COA analysis includes the 'war game' step, which is the most critical step within the COA analysis. In the war-game step, military staff members evaluate COA using various tools including DM&S systems. The DM&S system produces simulated battle outcomes through the interactions among the modelled entities in a simulated battle theatre. Because DM&S enables commanders to observe how the battle has been progressed with the given parameters, models, and scenarios, the commanders can anticipate how to plan their actions when the simulated progress is evident in the real world. This process of preparing COA with simulations is also known as *battle experiments* (Kass, 2006; Kass, 2008), and the battle experiment has become one of the key applications in the DM&S field.

One obstacle to the successful application of the battle experiments is the prolonged simulation time, which means that significant results from the battle experiments require a large number of simulations with a significant number of scenarios. For instance, our battle experiments of naval air defence scenarios (Kim *et al*, 2011a, b) require approximately 45 days just covering the limited parameter space in the model. This lengthy simulation time poses a challenge to our experimentation, as it includes a more elaborate model with a wider parameter space.

This paper aims to reduce the prolonged duration of battle experiments by adopting a simulation repository, which stores pre-simulated results using the tabulation technique (Bird, 1980) briefly described in Figure 2. This idea is particularly applicable to hierarchical modelling. Hierarchical modelling (Liu and Lee, 2000; Alur *et al*, 2003; Banerjee *et al*, 2004) represents a simulated world with the abstraction levels of simulation models. Such hierarchical modelling has been applied to analytic and simulation models. In particular, in the simulation models, the abstraction levels determine the resolution of the simulation details, thus the higher-level models handle the lower-level models, which describe the lower resolution of the simulated world. Then, the two models may interact hierarchically to feed scenarios and parameters back between the models. For instance, the high-level operation model in the naval air defence may depict the fleet-level operations, while the low-level operation model may portray the individual warship-level operations.

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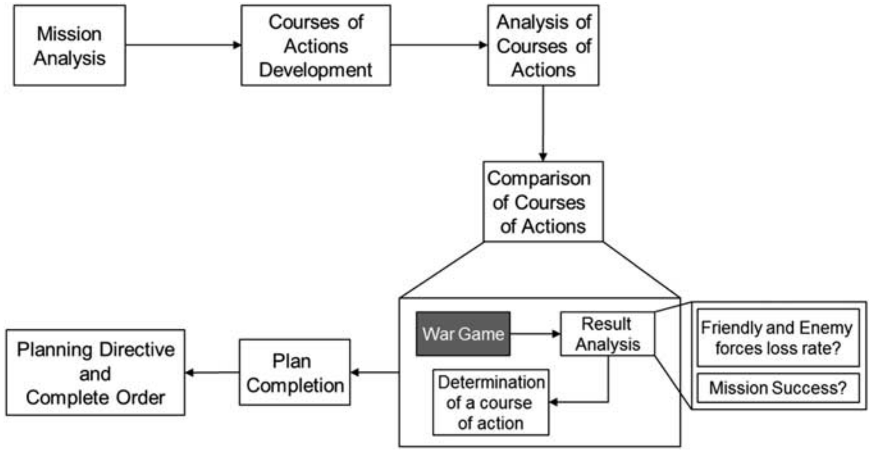


Figure 1 Military decision-making process of courses of action analyses with war games.

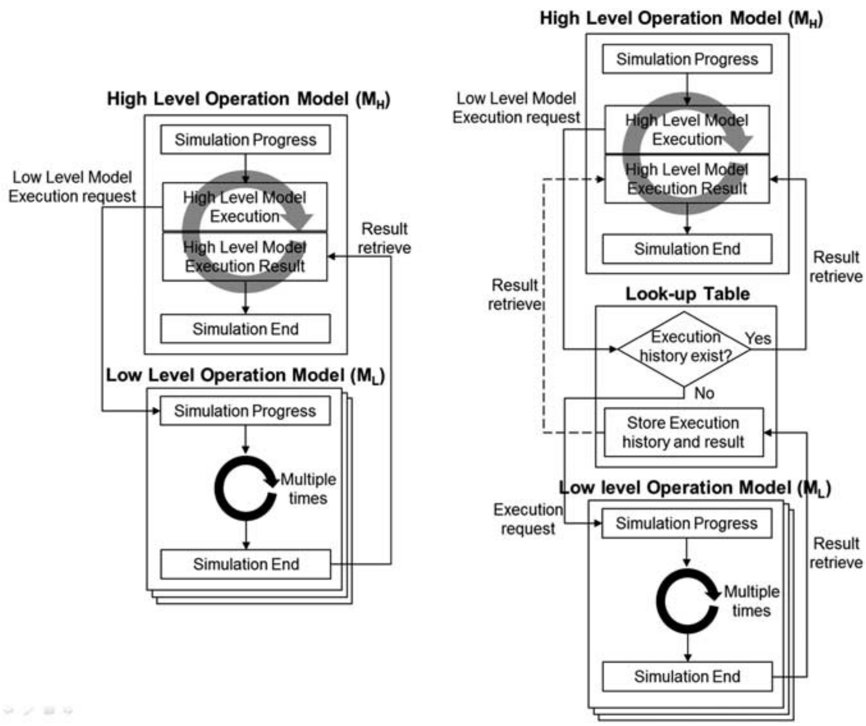


Figure 2 Hierarchical models execution process: (Left) without tabulation technique and (Right) with tabulation technique.

The high-level model feeds the scenario of engagements to the low-level models, such as when a fleet operation encounters an engagement between warships. When the engagements of the warship in the low-level models are over, each of the low-level model provides its simulation results to the high-level model; and the engagement results affect the next fleet operations. Our idea is to reduce the execution costs if a subset of the scenarios for these executions shares common aspects, particularly if the simulation result is reusable, such as in simulation models.

Specifically, we hypothesize that: (i) the number and, ultimately, the simulation time of executing hierarchical models will be decreased with the availability of the stored previous results, and (ii) this compressed execution will result in a limited degree of error in the experiment results, compared to the full execution. Whereas the tabulation technique is a well-known approach utilized to reduce the cost of computing by storing pre-computed results, this paper introduces a particular version of the tabulation technique that is adapted to the DM&S context. In the application, we augmented the commonly known tabulation technique with the quantization technique that assumes the interpolation of the simulated results. This augmentation allows us to reuse the simulation results by matching the parameters with pre-determined deviations; on the other hand, such allowance could lower the accuracy of the overall simulation results.

To investigate this trade-off, this paper evaluates the tabulation technique-based battle experiments by applying the approach to a naval air defence simulator. The simulator consists of hierarchical models at the mission level and at the engagement level (Piplani *et al*, 1994; Committee, 1997); the models are described in Kim *et al* (2011a, b). We compare the simulation times between the hierarchical models with and without the tabulation technique, and the hierarchical model with the tabulation technique speeds up the simulation up to seven times compared with the one without the tabulation technique. Also, the results from the tabulation technique-based experiments show the maximum deviation of 1.11% from those without the tabulation technique.

Furthermore, because the quantization technique ultimately reduces the number of result sampling, the accuracy of the results would be lowered; particularly, the variance would increase. This sampling effect is measured by two one-sided tests (TOST), *F*-test, ANOVA test, and Kruskal–Wallis test. The statistical tests represent that the sampling effect does not significantly affect the accuracy of the simulation results from the quantization technique compared to those from the full simulation runs.

We expect that the proposed technique is applied to accelerate simulations of hierarchical models with the allowable errors.

2. Background

This paper introduces the tabulation technique, which is well known in the general context of recursive programs. First, we compare tabulation techniques to our adapted tabulation technique in the DM&S context. Second, we discuss how to efficiently utilize the tabulation technique mechanism in the DM&S domain with a particular focus on the hierarchical modelling framework.

2.1. Previous researches using tabulation technique

Storing pre-computed results in the general context of recursions is discussed in Bird (1980). The research shows that computing a value by a function and a parameter once, at most, and storing the value for later usage reduces the cost of re-computing the function and the parameter, which is the fundamental principle of dynamic programming. For the storage of the values, the aforementioned research suggests utilizing a look-up table that is composed of the used parameters and functions as keys, and the computed values as values. In electrical engineering, a complicated function needs to be recalculated with various parameter sets in order to obtain an optimum structure for micro-strip leaky-wave antenna (Lin *et al*, 1996). Thus, the tabulation technique is used to avoid redundant calculations when the parameter sets match. In computational chemical engineering, a similar tabulation technique is used to reduce the calculation of the chemical compositions and the reactive flows because these compositions and flows match multiple times (Pope, 1997; Veljkovic *et al*, 2003; Zhang *et al*, 2005).

To the best of our knowledge, such tabulation techniques had not been used previously in DM&S in attempts to reduce simulation execution time. In addition, reusing pre-simulated results is not considered due to the diversity present in the battlefield situation, which reduces the chance of receiving exact matches of parameters used in previous simulations. However, we assume that simulations with not exact but similar parameters may share results through interpolations. We add this interpolation for similar parameters to the tabulation technique to make our contribution novel.

2.2. Potential use of the tabulation technique in hierarchical modelling

A large and complex system could be modelled with diverse considerations, which means that multiple models with various abstract levels exist in the complex model according to the various concerns in the complex model. To model such complex systems, hierarchical modelling could be utilized. Hierarchical modelling describes a complex system as a simple model containing multiple component models. With this hierarchical modelling, modellers could take advantages in developing complex models, such as developing models with ease, increasing the reuse of models, and aiding in model validation (Sargent *et al*, 1993). With these benefits, hierarchical modelling has been applied to analytic models (Asmussen and Glynn, 2007) and simulation models (Kim *et al*, 2011b). Although hierarchical modelling enhances the details of the models, hierarchical modelling has an impediment in the prolonged execution time. However, there are no studies, to our best knowledge, regarding the reduction of the execution time of hierarchical models in both analytic and simulation model cases. The defence system is representative of a complex system, thus DM&S generally applies to hierarchical modelling structures. Because many experiments with many scenarios and multiple models with various abstraction levels are required in DM&S, this research could support a faster analysis of the effectiveness of COA, and the tabulation technique could culminate in the reduction of the execution time of hierarchical models in DM&S.

To maximize the utilization of the tabulation technique in DM&S, many candidates (entries) are required in the table, particularly those that entail many times of simulation executions. In DM&S, there are a couple of examples of such a large number of simulation executions. One example is the data farming project in Project Albert (Brandstein *et al*, 2000). Data farming is a project that involves executing a simulation model more than one million times in order to yield the response surface of a large parameter space. However, the data farming did not include the use of the tabulation technique idea because the used model is not hierarchically structured to our knowledge. On the other hand, if the online hierarchical modelling technique is used to cover such large parameter spaces, then the tabulation technique idea would have resulted in efficient simulation runs by reusing previous simulation runs.

Our tabulation technique is most utilizable when models are interoperating and executed multiple times hierarchically. Hierarchical modelling is the use of hierarchical models through interoperations and data exchanges. This modelling approach is used for various reasons. For example, multi-resolution modelling, or MRM (Davis, 2000; Davis, 2005), is a type of hierarchical modelling. MRM uses a high abstraction model for the less important simulation period, and MRM simulates the details of scenarios by using a low abstraction model when it comes to the more important simulation period. This requires multiple switches between the low abstraction and the high abstraction models and data retrievals between the models as well. If MRM is used in those battle experiments that require a large number of scenarios and replications, then the data retrieval becomes a major factor in the total simulation execution time. Therefore, we expect that the tabulation technique would be appropriate in accelerating the executions by retrieving the simulation result of the lower abstraction model and feeding the stored result to the higher abstraction model.

Another example of hierarchical modelling is the interoperation of heterogeneous models for battle experiments. When analysing anti-torpedo operations (Seo *et al*, 2011), modellers develop a scenario that covers destroyers, torpedoes, submarines, and decoys, as it spans both engagement and engineering layers of the operations. Frequently, in the case of modelling such complex operations, modellers may decide to focus on either the engagement models or the engineering models of the weapons because of the modelling simplicity, the modelling scope, and available data. The case of the anti-torpedo operations segments results in the modelling of the mechanical movements and operations by an engineering model and the modelling of the tactical decision-making and deployment by an engagement model. Because these models are developed to focus on two different layers of operations, using the two models jointly through interoperations may increase the usability of the simulators (Sung *et al*, 2009). In this case, the engagement model will repeatedly require the simulation result of the engineering model. We expect that the tabulation technique would be useful in speeding up the runs because the simulation result from the engineering model can be stored and reused.

3. Overall architecture of the simulator with the tabulation technique

As we review the potential use of this technique, the proposed battle experiments using the tabulation technique assume two experimental conditions. First, the tabulation technique is applicable to the simulation of hierarchical models that exchange their information. Second, the tabulation technique is most utilizable when the battle experiment explores a large set of scenarios with a large parameter value set. These assumptions lead the design of the simulator architecture that optimally utilizes the tabulation technique.

Figure 3 illustrates the simulator architecture with the look-up table using the tabulation technique. Due to the assumption of hierarchical modelling, the architecture shows two models, which do not lose the generality of multiple hierarchical models, exchanging their information via the look-up table. Specifically, the hierarchical modelling is structured to allow for a high-level operation model, in the middle of its own execution, to execute the other model (or a low-level operation model) and to retrieve the simulated information from the low-level model.

In addition to the system architecture, Figure 3 describes the simulation execution flow using the tabulation technique. Without the tabulation technique, the low-level operation model is executed multiple times, as the high-level operation model sets up the execution parameters dynamically. Given this simulator without the tabulation technique, the number of the executions at the low-level operation model becomes very large because of the second assumption of the large parameter value set in the high-level operation model. Therefore, we place the look-up table on the information path between the high-level operation model and the low-level operation model. The look-up table stores the previous execution history of the low-level operation model, and when the high-level operation model requests an execution of the low-level operation model, the look-up table returns a stored piece of information if the request has already been made in the previous execution history. This approach saves execution time on the part of the low-level operation model. At the same time, this approach may suffer from the potential error because of two reasons. The first potential reason is the inexact match between the index key of the saved result and the index key of the run request. The second potential reason is the unexpectedly increased variance in the result values because we have reduced the simulation runs by reusing the saved result.

To further explain the look-up table with the hierarchical models, the sub-sections introduce the following: (i) the working mechanism of the look-up table using the tabulation technique, and (ii) the approximated match of the look-up table entries.

3.1. Operation mechanism of tabulation technique

After formally identifying the necessity of information exchange in hierarchical modelling, we developed the tabulation technique to assist the exchange of information regarding the variables. We explain the tabulation technique from two perspectives: the operation and the data structure of the look-up table.

Figure 4 shows the operation flow of the look-up table. The look-up table is only activated in the event of information exchange. Algorithm 1 specifies how the look-up table deals with information exchanges. When an exchange occurs, the look-up table checks whether the information requested, or the simulation result retrieval by the high-level operation model (M_H) from the low-level operation model (M_L) is stored in the data structure of the look-up table. If the result is not stored in the look-up table, the table executes M_L to produce, store, and deliver the requested result. If the result is already stored, the look-up table returns the result without executing M_L again.

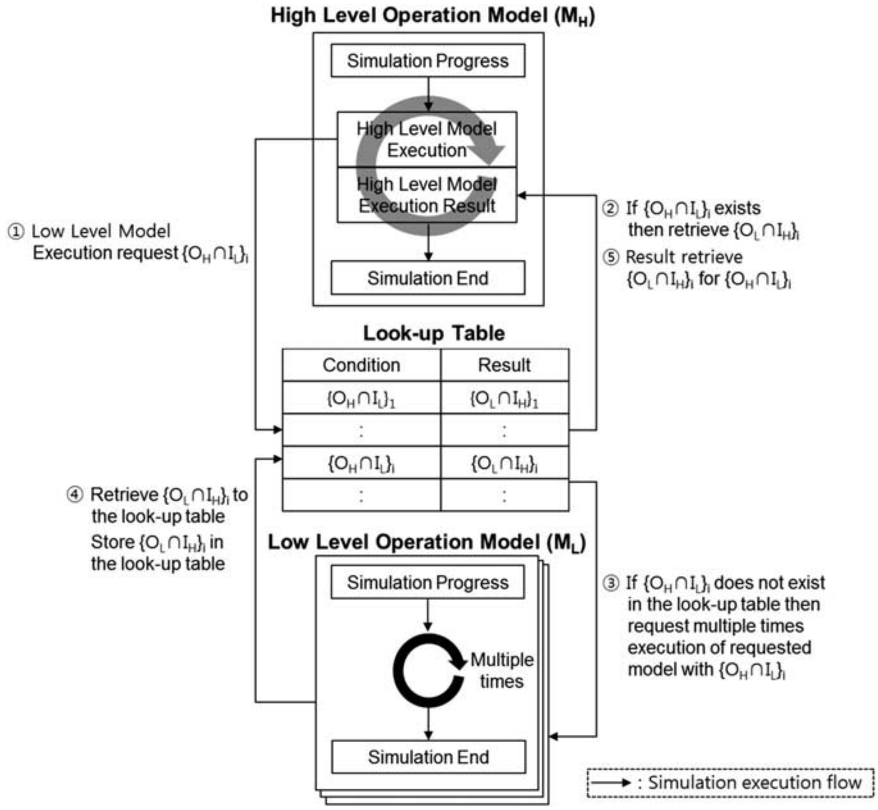


Figure 3 Simulator architecture using tabulation technique.

Algorithm 1 Tabulation technique in hierarchical modelling

Input: an element from the intersection set between output variables from M_H (O_H) and input variables from M_L (I_L), $V_I = O_H \cap I_H$

Output : an element from the intersection set between output variables from $M_L(O_L)$ and input variables from M_H (I_H), $V_O = O_L \cap I_H$

Set *conQ* to a set of *conditions*, where *condition* $\in V_I$
Set *resQ* to a set of *results*, where *result* $\in V_\eth$
Set R to a set of (v_i, v_o) relations, where $v_i \in conQ$ and $v_o \in resQ$
Begin
  when receives a request with $v_i \in V_I$
    if v_i exists in *conQ* **then**
      find a relation r associated with v_i in R
      $v_O \leftarrow$ retrieve a result in *resQ* using v_o in r
      result v_o
    else
      send a request with v_i to M_L
      wait until receiving a *result* from M_L
      insert the *result* to *resQ*
      insert v_i to conQ
      add (v_i, *result*) to R
      return *result*
  end
End

The tabulation technique system has the look-up table consisting of conditions and results, or keys and values. When it comes to hierarchical modelling, the potential variable exchanges are twofold in nature (Kim *et al*, 2011a, b). The first variable exchange shares information from M_H to M_L, and this exchange aims to run M_L by the scenario generated by M_H. The exchanged variables or the generated scenario is the output of M_H and the input of M_L, which is expressed as $O_H \cap I_L$. The second variable exchange sends information from M_L to M_H, and this exchange aims to continue M_H by the parameter provided by M_L. Therefore, the exchanged variables, or the provided parameters, are the input of M_H and the output of M_L, which is expressed as $I_H \cap O_L$. In our tabulation technique, $O_H \cap I_L$ and $I_H \cap O_L$ become conditions and results—or input and output, respectively. Generally, determining which will be the conditions and which will be the results depends on which model is primarily driving the simulation, as well as which model performs consecutive runs by the main simulation procedure. In Algorithm 1, we designed the condition to be the specification of executing M_L from M_H by setting M_H as the main simulation model in this hierarchy.

3.2. Approximated match of conditions

Given the operation mechanism and the look-up table, the tabulation technique requires matching the requested execution

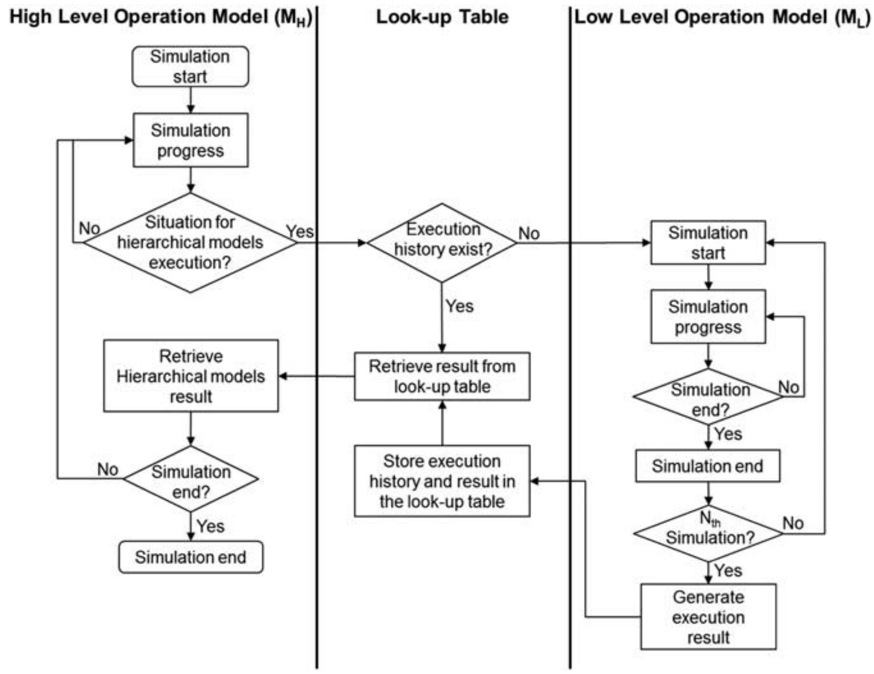


Figure 4 Simulation running procedure using tabulation technique.

to the stored conditions in the look-up table. When we adapt the tabulation technique to the DM&S field, we define a set of simulation parameters as a condition and, eventually, a key in the look-up table. Therefore, the key matching in the look-up table needs to be extended to cope with the nature of the simulation parameters. The simulation parameters are often typed as either categorical or numeric. The categorical parameter, such as either *on* or *off, treatment* or *non-treatment*, is easy to be matched exactly. However, the numerical parameter is frequently impossible to be matched precisely, particularly when the range of the value type is in the real-number dimension. Hence, we adopt a method similar to quantization in the signal processing field (Sayood, 2012). Quantization is the process of converting a continuous set of values to a discrete set, which acts as an interpolation technique in this application. What follows is our definition of the quantization in our tabulation technique system.

$$V_{Quantized} = \left\lfloor \frac{V}{\alpha} \right\rfloor \times \alpha \tag{1}$$

Formula (1) shows the quantization of a numeric parameter, V, so that the numeric parameter is turned from a continuous value into a discrete value. If a parameter value from the request and a value from a condition fall into the same discrete value, the two parameter values are matched in our tabulation technique system. In Formula (1), the width of the quantization interval, or α, determines the accuracy level of condition matching after the quantization. If α becomes larger, the width of quantization becomes wider, accepts more deviation among values, and contains more noise in the parameter. Therefore, unless α becomes extremely small, this matching procedure of the numeric parameter will contain errors from the condition matching process, as well as eventually in the result, to a certain extent. Therefore, we analysed the change in the error level in accordance with the quantization width from the mean and the variance perspectives, statistically.

4. Case study: hierarchical modelling of naval air defence

The proposed tabulation technique is best applicable to a battle experiment with hierarchical modelling. We evaluated the performance of the tabulation technique by applying it to a hierarchical modelling case on the naval air defence. The naval air defence is the activity aimed at neutralizing enemy warplanes and missiles that are approaching friendly warships. The defence is a joint effort between the command and control (C2) personnel, the detection system, and the weapon system of the warships (Carley, 1995; Nearly, 2008; Interviews, 2010). In a defensive activity, the detection system (ie, radar onboard) warns the C2 personnel when threats—such as enemy aircraft and missiles—are detected. Subsequently, the C2 personnel assigns which weapon system will intercept the incoming threats. Finally, the weapon systems (ie, surface-to-air missiles [SAM] and closed-in weapon systems [CIWS]) are fired upon the threats.

From our previous work, we have two models that focus on two different layers of this defensive activity. First, we have a mission-level model depicting the detection alert by the radar, the engagement order by C2, and the interception by the warship (see

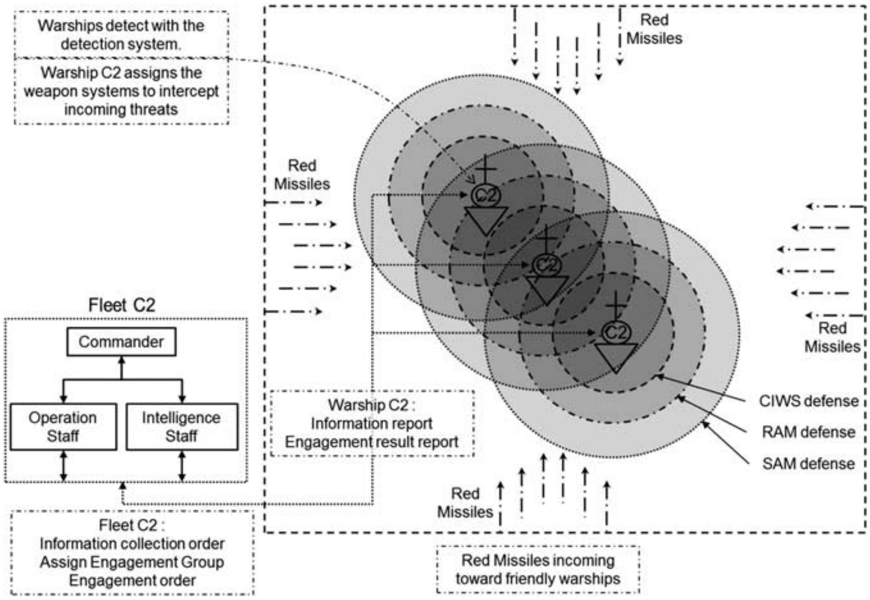


Figure 5 Scenario description for naval air defence.

Figure 5 for the scenario description and Table 1 for the modelled information). Second, we have an engagement-level model describing the details of the interception by the warship. The details include three different weapon systems—SAM, Rolling-Airframe Missile (RAM), and CIWS—all covering the estimated trajectory of the missiles (see Figure 5 for the scenario and Table 1 for the model inputs and outputs). The first model simulates the higher level of the activity, such as the decision-making and the platform movements at the high abstraction. The second model simulates the lower level of the activity, such as the mechanical details of the trajectories.

The naval air defence simulator consists of these two models, both of which are involved in hierarchical modelling. Figure 6 shows the simulation progress of the hierarchical modelling. When the simulation starts, the mission-level model simulates the trajectory of the incoming missiles. As soon as the missiles are within the detection range of the warships, C2 decides how to assign the missiles to the warships for their interceptions. After the assignment, the mission-level model requests execution of the engagement-level model to retrieve the result of the interceptions by weapon systems. Next to the retrieval of the interception result, the mission-level model continues to the next round of interceptions or it terminates the simulation.

We apply our tabulation technique by assuming the mission-level model as M_H; and the engagement-level model as M_L. Greyed variables in Table 1 are overlapping parameters and candidates for keys of the look-up table. For example, *engagement ratio* in Table 1 is an input variable of the engagement-level model and an output variable of the mission-level model as well. It means *engagement ratio* can be used as shared information, which means *engagement ratio* can be used as the key of the look-up table. In our application, the key of the look-up table becomes the input variables to the engagement-level model, such as *warship move, red missile move*, and *engagement ratio*, generated from the mission-level model (see the overlapping parameters of I_L and O_H in Table 1). Generally, there are two different types of overlapping parameters or keys of the look-up table. The first type is the parameter of engagement rates (ie, *engagement ratio*) that specifies the number of warships and the number of anti-ship missiles. The two numbers are integers, and due to the limited range of the numbers, they are exactly matched in our look-up table. On the other hand, the second type of parameter is the positions of warships and missiles (ie, *warship move* and *red missile move*), and these position parameters are continuous and difficult to be matched exactly. Therefore, for this second type of parameter, we apply the quantization method discussed earlier. To apply the quantization method, we need to narrow down the number of position values—such as the *X, Y*, and *Z* coordinates of warships and missiles—to a single parameter. Therefore, we designed a metric that compresses the coordinates into one value, which is the average distance.

In the naval air defence scenario, the relative positions between warships and missiles are critical. Which missile is close to which warship dictates which case should be prioritized. Hence, we tracked the relative distances between the warships and missiles. These relative distances are calculated for every possible pair of a warship and a missile. For instance, if we have *N* warships and *M* missiles, we have *N***M* relative distances. Then, we average the relative distances to compress the information into a single value. This is the rationale behind the average distance. This average distance is used as a key in the look-up table to identify

Table 1 List of parameters and performance indices for the engagement-level and the mission-level models of naval air defence simulation

I_L	I_H	O_L	O_H	*Name*	*Description*
O	*O*			Number of Warships	The number of warships composing fleet (default = 3)
O	*O*			Number of Red Missiles	The number of red missiles in the threat forces (varied in the experimental design)
	O			Warship Position	The *X*, *Y*, *Z* coordinates of warships composing fleet (warships' default coordinates = < 150 km, 170 km > , < 160 km, 170 km > , < 170 km, 150 km >)
	O			Red Missile Position	The directions of red missiles in the threat forces. The red missiles are located in four directions relative to warships before engagement starts. For each direction, the number of red missiles is same (four default directions = North, West, East, South)
O			*O*	Warship Move	The *X*, *Y*, *Z* coordinates of warships generated from the engagement level
O			*O*	Red Missile Move	The *X*, *Y*, *Z* coordinates of red missiles generated from the engagement level
O			*O*	Engagement Ratio	The ratio of the number of warships to the number of red missiles generated from the mission-level model
O	*O*			Red Missile Speed	The missile speed of red missiles in the threat forces. The speed of all missiles is the same (varied in the experimental design)
O				RADAR Radius	The maximum detection radius of RADAR on warship (default = 1000 km)
O				CIWS Radius	The maximum radius of CIWS warship (default = 3.2 km)
O				RAM Radius	The maximum radius of RAM on warship (default = 10 km)
O				SAM Radius	The maximum radius of SAM on warship (varied in the experimental design)
	O			C2 Decision-Making Time	The C2 decision-making time (second) (varied in the experimental design)
	O			C2 Decision Radius	The C2 maximum radius of engagement decision (varied in the experimental design)
			O	Survival Rate(SR) of Warships	The ratio of the number of survival warships to the total number of warships when an interoperation simulation is terminated
	O	*O*		Single Engagement Average Intercept Rate	The average for ratios of the number of intercept missiles to the total number of missiles for *N* replications

I_L indicates the input variables of the engagement-level model; I_H is the input variables of the mission-level model; O_L is the output variables of the engagement-level model; and O_H is the output variables of the mission-level model. Grayed-variables are overlapping parameters and thus candidates for keys of the look-up table.

similar engagement positions pertaining to the warships and missiles. We may enhance this representative metric to better compress and preserve the position information in our future research. Yet, we also statistically showed that this metric is good enough to identify the similar engagement positions from the keys in the look-up table. Figure 7 shows the illustrative scenario of the average distance calculation.

5. Evaluations for simulation time and accuracy

The success of the tabulation technique in the DM&S framework relies on two evaluation criteria. The first criterion is obtaining a significant reduction in simulation time. The second criterion is achieving satisfactory accuracy in the reduced samplings of scenarios. We evaluated the two criteria through battle experiments using the tabulation technique in our naval air defence case.

5.1. Battle experiment design

Given the case study, we designed a simulation experiment to observe the changes in the survival rates of the warships by varying five factors related to the naval air defence (refer to Table 2). This experiment design requires executing the hierarchical model 2430 times (ie, 243 cells by 10 times). From the evaluation perspective, the tabulation technique system should shorten the simulation experiment time and minimize the difference in the survival rate between the simulators, with and without the tabulation technique. It should be noted that the experiment design uses the full-factorial design, and each replication has no interference with the other replication. Additionally, the simulation model does not change by the experimental cell, thus the results have the same distribution when they are drawn from the same experimental cell. From these characteristics, we claim the characteristics of independent and identically distribution (IID) among the results, and IID is assumed in every statistical analyses in the section of *Simulation accuracy evaluation*.

Therefore, the evaluation relates to the comparisons in terms of simulation time and accuracy between the experiment with and without the tabulation technique. The proposed tabulation technique is able to increase the hit ratio by increasing the quantization width, or α, applied to the average of the relative distances. We experimented with the simulation time reduction and the accuracy loss as we varied the quantization width by 1, 10, and 100 on the average distance in the scenario of the lower-level operation model. Additionally, the below is the system specification that we experimented.

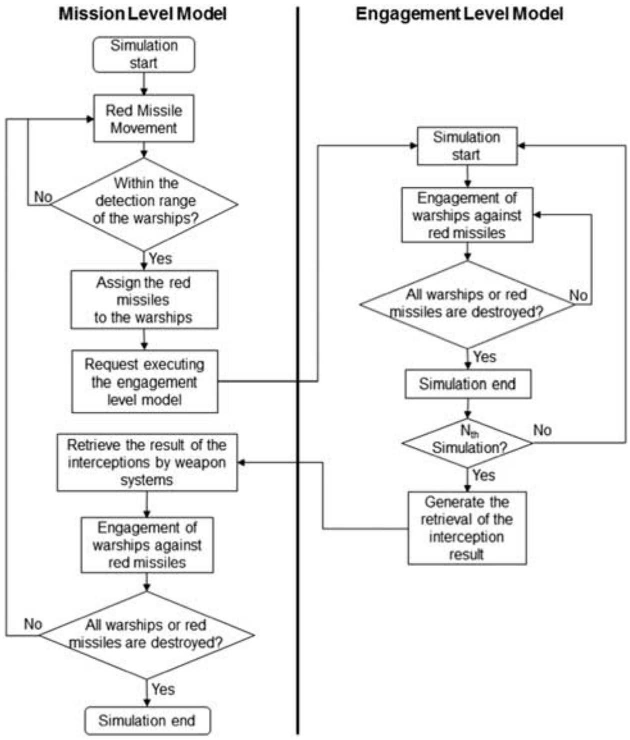
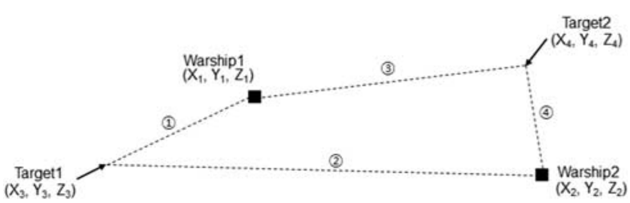


Figure 6 Simulation progress of the hierarchical models for the naval air defence simulator.



Name	Value
# of warships	2
# of targets	2
Average Distance	$\frac{①+②+③+④}{\text{\# of warships} \times \text{\# of targets}}$
Average IR of targets	$\frac{IR_{Target1} + IR_{Target2}}{\text{\# of Targets}}$

Figure 7 Compression of positions into average of distances.

- OS: Window 7 Enterprise (64-Bit)
- CPU: Intel Core i7, 2.79GHz
- RAM: 6.00GB

5.2. Simulation time evaluation

The hierarchical model without the tabulation technique (original in Figure 8) takes approximately 35 879 min, or 24.9 days, to finish 2430 simulation runs, while the hierarchical model with the tabulation technique takes approximately 5960 min, or 4.1 days, to finish the runs when the quantization width is one (see Figure 8). The battle experiments without the tabulation technique have already been introduced in our previous work (Kim *et al*, 2011a, b). This runtime points to an execution that is approximately six times faster in terms of simulation time. Moreover, to see the speed-up performance with varying quantization widths, we calculated speed-up ratio of quantization width 1, 10, and 100. Speed-up ratio is defined as the simulation runtime of the quantization width over the simulation runtime of the original case. Figure 9 illustrates that the speed-up ratio is proportional to the quantization width. Moreover, Figure 9 indicates that the increase of the speed-up is smaller, as the quantization width is getting larger.

To investigate the simulation time reduction further, we observed the hit ratio and the size of the look-up table. First, we suspected that the simulation time was reduced, due to increasing reuses of previously stored experimental results in the look-up table. This is confirmed by observing the high tabulation technique hit ratio. For instance, in Figure 10, when the quantization width is 100, the look-up table hit ratio reaches 84.2%, which means a simulation request from M_H to M_L is already stored in the look-up table with such a high probability. Furthermore, Figure 10 represents that the hit ratio is proportional to the quantization width, which results in the high speed-up performance of the quantization width 100. Further evidence of the reuse is the size of the look-up table in Figure 11. Over the course of running the 2430 simulations, the look-up table identifies 474 distinct request conditions in the quantization width 100.

Table 2 Battle experiment designs of the standalone engagement model, the standalone mission-level model, and the interoperation model
Ten simulation replications are done for each experiment cell. The experiment variables are selected factors to gain the insights into the doctrine.

Experiment variable name	*Experiment design*	*Implications*
Number of Red Missiles (I_L, I_H)	8, 16, or 24	Low, medium, or high level of air threats
Red Missile Speed (I_L, I_H)	300, 480, or 680	Slow, medium, or fast speed of red missile (m/s)
Missile Interception Radius of a Warship SAM (I_L)	90 000, 110 000, or 130 000	Short, medium, or long interception radius of warship (m)
Decision Radius of Fleet C2(I_H)	90 000, 110 000, or 130 000	Short, medium, or long decision radius of fleet C2 (m)
C2 Decision-Making Time (I_H)	180, 200, or 220	Fast, medium, or slow decision-making timings (second)
Total Number of Battle Experiment Cells	243cells (=3 × 3 × 3 × 3 × 3 cases)	Total number of battle experiment cells according to experiment variables

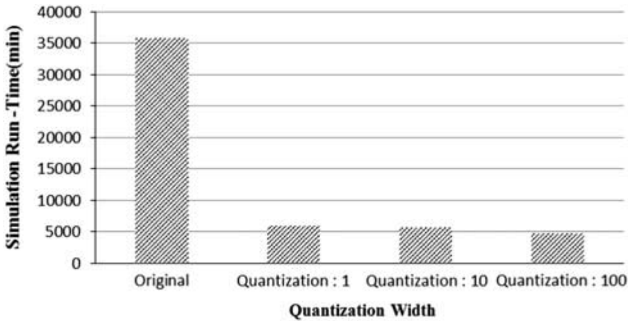


Figure 8 Simulation time according to the quantization width.

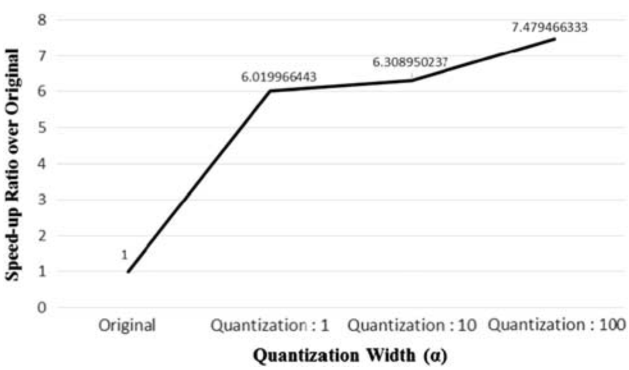


Figure 9 Speed-up ratio varying quantization width (α) over original.

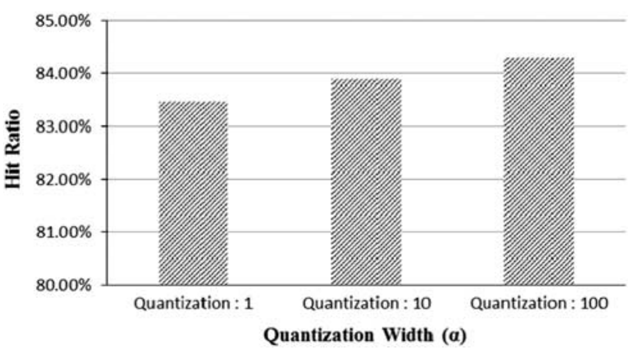


Figure 10 Hit ratio according to the quantization width.

5.3. Simulation accuracy evaluation

From the previous section, we discovered that the quantization width affects the reduction of the simulation time. As we increased the quantization width, the tabulation technique matches keys with increased tolerance. As the matches become easier, the look-up table will have fewer entries, greater hit ratios, fewer additional executions, and less simulation time, sequentially. Having mentioned this simulation time reduction, we needed to verify whether the accuracy of the simulation is negatively impacted by the simulation time reduction. To verify this, we compared the results from the simulation executions without the tabulation technique, called as the original case, to the results from the executions with the tabulation technique specifying the quantization width as 1, 10, and 100.

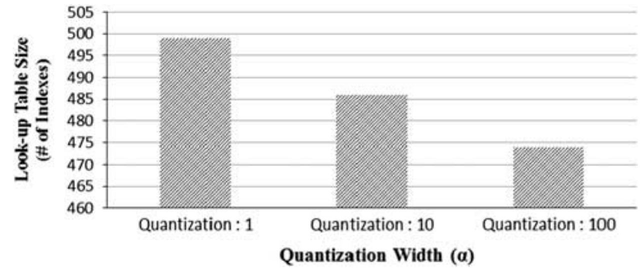


Figure 11 Look-up table size according to the quantization width.

Figure 12 shows the visual comparison between the results with and without the tabulation technique. For instance, the curve between the survival rate and the C2 delay time, which is the first graph in Figure 12, shows the increasing deviation of the results generated from the runs with the tabulation technique from the original result obtained from the full execution. The result from the quantization width one is almost identical to the original curve, whereas the curve from the quantization width 100 deviates far from the original curve. This deviation trend is consistently observed in the rest of the graphs in Figure 12.

For more statistical analysis for the accuracy, we evaluated standard errors of various experimental cases. In the experimental cases, we increased the quantization width as 1 (QW1), 10 (QW10), and 100 (QW100) for checking errors from the quantization width, and the number of the red missiles as 8, 16, and 24 for checking errors from the number of parameters. Figure 13 represents graphical views of the standard errors in the experimental cases, and Table 3 shows the average and the standard errors of the experimental results.

Figure 13 represents that the amount of the standard errors are proportional to the number of red missiles. For example, the error in QW1 and the number of the red missiles 24 is 0.014, while the error in QW1 and the number of red missiles 8 case is 0.011 (see Table 3). These results implied that the errors might increase, as the number of parameters increase. On the other hand, Figure 13 indicates that there were little changes in the errors when the quantization widths are increased, and the ratios in Table 3 show little changes in accordance with the increase of the quantization widths. Hence, these results indicated that the errors from the quantization width are not significantly large to influence the averages.

In addition to the visual confirmation (Figure 13) and the confidence interval analyses (Table 3), we performed various statistical tests to the experimental results. From the statistical tests, we can draw a conclusion that the two simulation results are statistically indistinguishable, if they have statistically identical values from two perspectives: the mean and the variance of the result distribution.

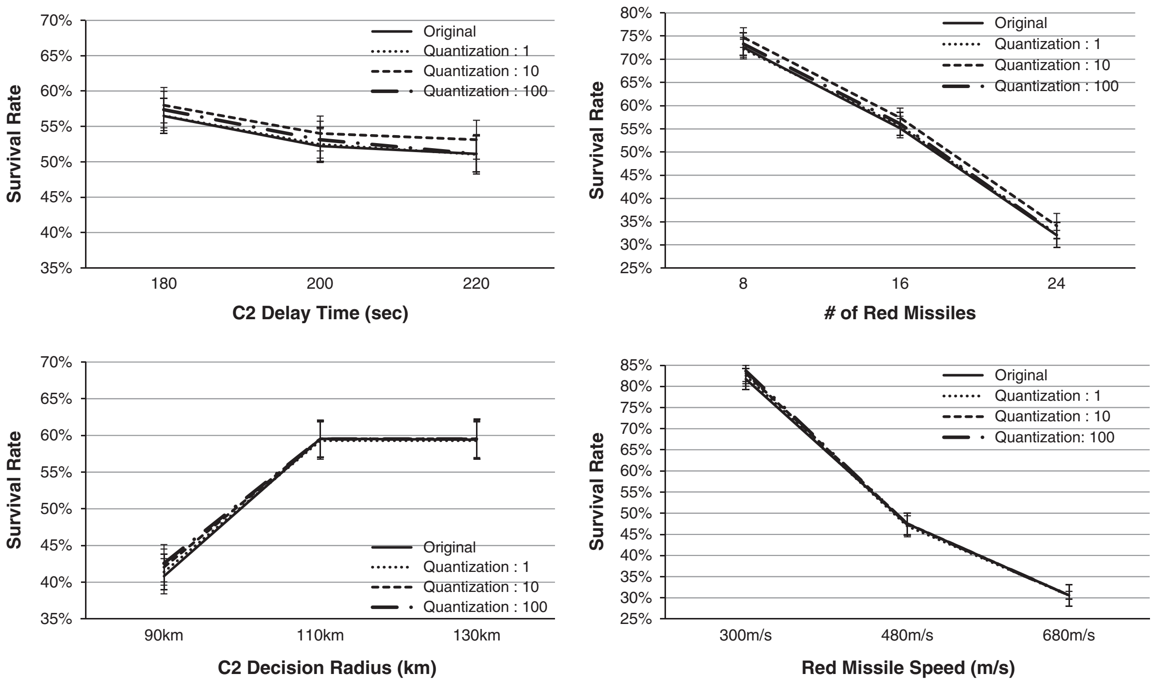


Figure 12 The performance changes in accordance with the quantization width and a single variable change of the performance indices.

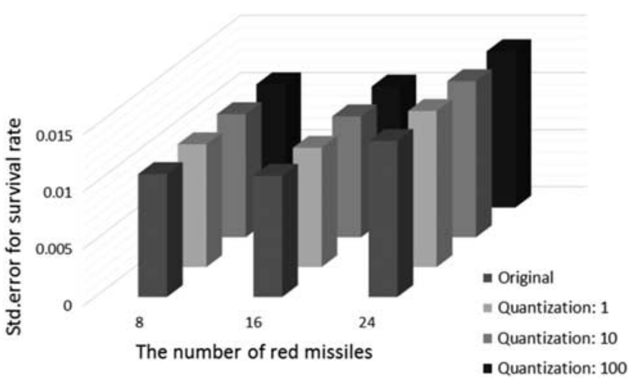


Figure 13 Standard error of the performance measure (survival rate) in accordance with the quantization width and the number of the red missiles.

To show the equivalence of means ($\mu_{baseline}$, μ_{QW}) from the baseline case (full experiments without quantized tabulation) and from the experimental cases (shortened experiments with quantized tabulation), we utilized the two one-sided tests (TOST) following the research from Schuirmann (1987) and Liu and Chow (1992). The ordinary *T*-test is not appropriate because the inability to reject the null hypothesis ($\mu_{baseline} = \mu_{QW}$) does not indicate the acceptance of the null hypothesis. Therefore, we utilize the below null hypothesis and the alternative hypothesis consisting of two *T*-tests.

$$H_0 : \mu_{baseline} - \mu_{QW} \leqslant \varepsilon_L \text{ or } \mu_{baseline} - \mu_{QW} \geqslant \varepsilon_U$$

$$H_1 : \varepsilon_L < \mu_{baseline} - \mu_{QW} < \varepsilon_H$$

The above hypotheses imply that the difference between the difference of the simulation results (ie, average survival rate of the warship simulation in this paper) from the baseline case and the experimental case significant falls into the range of ε_L and ε_U. We set this range as 0.01, which is the 1% of the survival rate range. In other words, if the null hypothesis is rejected, this confirms that the shortened simulation executions with tabulation produce simulation results with 0.5% difference at maximum with statistical significance.

Table 4 shows the results of TOST. In total, six *T*-tests were performed, which are three TOST tests for three corresponding quantization levels. The TOST was conducted in testing the equivalence of the warship survival rates. When we set the equivalence threshold to 0.5% (1% when we consider two sides) of the survival rate, the tests reject the null hypothesis of QW1 and QW10, but the null hypothesis of QW100 cannot be rejected. This indicates that statistically QW1 and QW10 yield significantly equivalent simulation results, but QW100 cannot produce the result from the baseline.

After we statistically evaluated the similarity of the mean values of the baseline and each quantized cases, we evaluated the similarity of variances by the pairs of two selected groups. Because our tabulation technique reduces the simulation time by performing fewer samplings from the stochastic results of the lower-level model, this small number of sampling may profoundly impact the increases of variance. To check the similarity of variances from the four cases, we applied *F*-test to their variances. In the *F*-test, the null hypothesis indicates the same variance from the two groups. The *F*-test results between the original and each quantized cases represent

Table 3 Average and standard error of the performance measure in accordance with increasing the number of red missiles and the quantization width (QW)

	Average of survival rate			*Std. error of survival rate*		
No. of red missiles	*8*	*16*	*24*	*8*	*16*	*24*
Original	0.737	0.567	0.324	0.011	0.011	0.014
QW1	0.737	0.558	0.322	0.011	0.010	0.014
QW10	0.739	0.561	0.322	0.011	0.011	0.014
QW100	0.739	0.561	0.322	0.011	0.011	0.014

Table 4 Two one-sided tests (TOST) with two null hypothesis for *T*-tests to analyse the equivalence of performance indices ($\varepsilon_L = -0.005$, $\varepsilon_U = 0.005$), Count = 2430 cases

	Survival rate	
	$H_0{:}\mu_{baseline} - \mu_{QW1} \leqslant \varepsilon_L$	$H_0{:}\mu_{baseline} - \mu_{QW1} \geqslant \varepsilon_U$
Mean	−0.00069	0.000686
Std. Dev.	0.057782	0.057782
t-stat	310.0657	−310.0657
p-value	0.000	0.000
	$\mu_{baseline} - \mu_{QW10} \leqslant \varepsilon_L$	$H_0{:}\mu_{baseline} - \mu_{QW10} \geqslant \varepsilon_U$
Mean	−0.00453	0.004527
Std. Dev.	0.072388	0.072388
t-stat	5.1536	−5.1536
p-value	0.000	0.000
	$H_0{:}\mu_{baseline} - \mu_{QW100} \leqslant \varepsilon_L$	$H_0{:}\mu_{baseline} - \mu_{QW100} \geqslant \varepsilon_U$
Mean	−0.00604	0.006036
Std. Dev.	0.485422	0.485422
t-stat	−8.4586	8.4586
p-value	1.000	1.000

Table 5 *F*-test results for performance indices ($\alpha = 0.05$)

	Survival rate	
	Original	QW1
Mean	0.550	0.551
Variance	0.118	0.116
F-stat	1.012	
p-value	0.460	
	Original	QW10
Mean	0.550	0.552
Variance	0.118	0.121
F-stat	0.980	
p-value	0.438	
	Original	QW100
Mean	0.550	0.557
Variance	0.118	0.119
F-stat	0.996	
p-value	0.488	

Table 6 ANOVA test results for performance indices ($\alpha = 0.05$)

Source	*SS*	*df*	*MS*	*F-ratio*	*p-value*	*Critical Value of F*
Between Groups	0.006	3	0.012	0.017	0.997	2.614
Within Groups	115.029	968	0.119			
Total	115.035	971				

p-value > 0.05 (see Table 5), which indicates that the null hypotheses cannot be rejected. Therefore, we can reject the alternative hypothesis: the different variances from the different groups. Though the number of the sampling is reduced by the quantization techniques, this suggests that the variances from original and each quantized techniques do not significantly differ within the confidence level of 95%. Having said that, it should be noted that this does not prove the equivalence, either.

To investigate the variance of the four cases even further, we perform the one-way analysis of variance (ANOVA) test by setting the treatments as the quantized cases. The null hypothesis of ANOVA is that the mean values from the tested groups are statistically identical, assuming the equivalent variances of the groups. Table 6 shows the results of the ANOVA test as *p*-value > 0.05, which means that the null hypothesis cannot be rejected within the confidence level of 95%. Hence, we can reject the alternative hypothesis of different means from different groups. Surely, this does not prove the equivalence of the means, either. However, from TOST in Table 4, we statistically observed the equivalence of means between the baseline, QW1, and QW10, which excludes QW100.

ANOVA test assumes both equivalent variances and normality of the results of the baseline and the quantized cases, which are not guaranteed by the analyses. To make up for this, we performed the Kruskal–Wallis test, which is a non-parametric method of ANOVA without equivalent variances and without normality. Therefore, the null and the alternative hypothesis are identical to the ANOVA, except the assumptions. The null hypothesis states that the means from the groups are identical. Table 7 illustrates the result of the Kruskal–Wallis test, and the test presents *p*-value > 0.05. This suggests the alternative hypothesis is rejected: we cannot confirm the significantly different means between groups. Again, this does not prove the equivalence of means, and we showed that QW1 and QW10 show the equivalent mean to the baseline.

To sum up the results of the statistical tests, Table 8 presents the summary of the simulation time and the accuracy evaluations of the tabular techniques. The baseline case, which does not use any tabulation techniques, requires a large amount of simulation time. Because some simulation parameters cannot be matched exactly, we defined the quantization width (QW) of key matching in the look-up table. By adopting the techniques, the simulation time is reduced by 6.02 times in QW1, by 6.31 times in QW10, and by 7.48 times in QW100. As we increase the quantization width, the matching becomes more frequent; therefore, simulation time is further reduced. However, as the quantization width increases, the accuracy or the proximity to the original result is reduced.

When we reduce the simulation time to 6.02 times by QW1, the mean of the result is almost same as that of the original run (0.001% deviation from the original), and from TOST, the equality is statistically significant with the error range of 1% survival rate. When we increase the speed up to 7.48 times, however, the result more deviates (1.110% deviation from the original), and the equality is not able to be accepted by TOST. Moreover, although there is no significant difference of the variances in the experiments, the fewer number of samplings by the tabulation techniques could incur the non-equivalence of the variances. There was no statistically significant evidence of equal variance between groups. Hence, an actual trade-off can occur between the simulation time and the variance equivalence, and when the simulation time trade-off gets bigger, the equivalence of means can be rejected, that is the case of QW100.

Table 7 Kruskal–Wallis test results for performance indices ($\alpha = 0.05$)

Survival rate					
Quantization level	*N*	*Mean rank*	*Chi-square*	*df*	*p-value*
Original	243	481.880	0.143	3	0.986
QW1	243	485.990			
QW10	243	486.990			
QW100	243	491.140			
Total	972				

6. Conclusion

The focus of this paper is to reduce the duration of battle experiments by adapting a tabulation technique when the experiments use a hierarchical modelling approach. We propose a framework with the tabulation technique in the DM&S context, and the performance of the tabulation technique is evaluated by applying it to a hierarchical modelling case in the context of the naval air defence. The adapted tabulation technique stores and reuses simulation results when one model requests that another model be executed in the hierarchical modelling simulation. This reuse of the simulation results is the key to reducing the prolonged simulation time. Additionally, relaxing the tolerance in the matching condition increases the reuse frequency and decreases the simulation time. Such relaxation, however, could lower the accuracy of the entire simulation, particularly from the variance perspective. This paper applies four statistical tests for the result accuracy to its application, and the test results show that there are no significant differences between the original results and the relaxed results from the mean and the variance perspectives. We expect that this paper is a step towards performing battle experiments with more detailed models, with diverse scenarios, and in less simulation time.

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Table 8 Integrated evaluation for simulation time and accuracy

Quantization Level	*Simulation time*		*Accuracy*			
	Wall-time duration	*Speed up*	*Mean*	*Deviation from original (%)*	*Mean comparison p-value (TOST, one side)*	*Variance comparison p-value*
Baseline	35 879 (min)	1 time	0.533			
QW1	5960 (min)	6.02 times	0.533	0.001	0.000	0.461
QW10	5687 (min)	6.31 times	0.537	0.002	0.000	0.438
QW100	4797 (min)	7.48 times	0.539	1.110	1.000	0.488

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