Defuzzification method for multishaped output fuzzy sets

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A defuzzification method is an important performance factor in fuzzy logic controllers (FLCs). When an FLC uses multishaped output fuzzy sets, existing defuzzification methods have the fat shape dominance phenomenon. The authors propose a new defuzzification method for multishaped output fuzzy sets. The method takes the degree of certainty on the fired level into account in the defuzzification process. Simulation experiments with a nonlinear plant show that an FLC employing the proposed defuzzification method is more stable and robust than existing FLCs.

Introduction: In a fuzzy logic controller (FLC), the final control outputs depend not only on the rules used but on the inference and defuzzification method [1]. Also, the shapes of output fuzzy sets have a great effect on the output of defuzzification. However, when an FLC uses multishaped output fuzzy sets, the problem of the fat shape dominance phenomenon may arise [3].

Yu *et al.* [3] proposed an inference method, and Hellendoorn *et al.* [2] introduce a new defuzzification method for multishaped output fuzzy sets. The Hellendoorn method depends only on the area of the clipped output fuzzy set regardless of the degree of certainty on the fired level.

We develop a new defuzzification method which takes into account the degree of certainty on the fired level, namely, α-cut. The Hellendoorn method is a special case of our defuzzification method. This fact indicates that our method can be viewed as an extension of his method. The simulation results show that our defuzzification method is more stable and robust than the Hellendoorn method.

Fat shape dominance phenomenon: The problem of the fat shape dominance phenomenon [3] is summarised as: when an FLC uses output fuzzy sets which have different or asymmetrical shapes, the final FLC outputs are mainly influenced by the fat shape output membership element even though the membership is fuzzier than the slim shape element.

If the rules of an FLC are inconsistent, then this phenomenon becomes more severe.

New defuzzification method: First, we define the measure of certainty which is used as a basis of our defuzzification method.

(i) Definition 1: Measure of certainty of α*-cut:* Let $X \subset R^1$ be the universe of discourse and $x \in X$. Let F_X be a family of fuzzy sets on X and $\acute{A} \in F_X$, and let $w_{\acute{A}}(\alpha)$ be the support length of α-cut; the measure of certainty of α-cut is then defined as

$$m_c(\alpha) = \frac{\alpha}{w_{\acute{A}}(\alpha)} = \frac{\alpha}{\mu_{\acute{A}}^{-1}(\alpha)_{max} - \mu_{\acute{A}}^{-1}(\alpha)_{min} + 1} \quad (1)$$

where $\mu_{\acute{A}}^{-1}(\alpha)$ is the inverse membership function of $\acute{A}$.

Hellendoorn *et al.* [2] propose a defuzzification method to solve the fat shape dominance phenomenon for multishaped output fuzzy sets. Eqn. 2 shows his defuzzification method:

$$u^* = \frac{\sum_{i=1}^{n} \frac{s_{im}\alpha_i^2}{area(S_i)}}{\sum_{i=1}^{n} \frac{\alpha_i^2}{area(S_i)}} \quad (2)$$

where α_i is the value with which the rule R_i fires, $s_{im} = \mu^{-1}(1)$ and S_i is the clipped output of the fired rule. His method does not consider the degree of certainty of output fuzzy sets on the level of α-cut.

On the basis of the discussion, we propose a new defuzzification method which depends on the degree of certainty on the level of α-cut. We call the method the level grading method (LGM). If an FLC uses n output fuzzy sets, then the LGM is given as follows:

$$u^* = \frac{\sum_{i=1}^{n} s_{im} m_c(\alpha_i)}{\sum_{i=1}^{n} m_c(\alpha_i)} = \frac{\sum_{i=1}^{n} \frac{s_{im}\alpha_i}{w_{A_i}(\alpha_i)}}{\sum_{i=1}^{n} \frac{\alpha_i}{w_{A_i}(\alpha_i)}} \quad (3)$$

where α_i is the value with which the rule R_i fires, $s_{im} = \mu^{-1}(1)$, and $w_{\acute{A}}(\alpha_i)$ is the support length of α-cut.

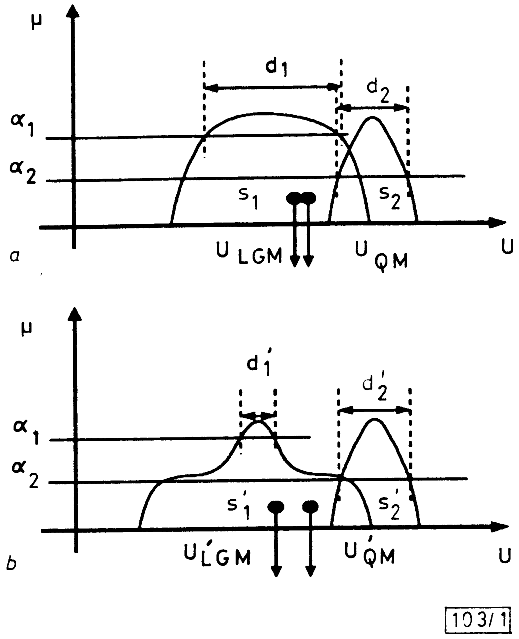

Fig. 1 Comparison between LGM and QM

Conditions:
$s_1 = s_1'$
$s_2 = s_2'$
$d_1 \neq d_1'$
$d_1 = d_1'$
Conculsions:
$U_{QM} = U'_{QM}$
$U_{LGM} \neq U'_{LGM}$

Fig. 1 shows the comparison of two methods, i.e., QM and LGM. As we intuitively expect, the output of defuzzification in Fig. 1*b* should be near the left membership function because the degree of certainty of the left membership function on the fired level is larger than that of Fig. 1*a*.

As a special case, if all output fuzzy sets have rectangular shapes, then the support length on the all level of α-cut becomes constant. Finally our method becomes the same as the QM as shown in eqn. 4.

$$u^* = \frac{\sum_{i=1}^{n} \frac{s_{im}\alpha_i}{w_i}}{\sum_{i=1}^{n} \frac{\alpha_i}{w_i}} = \frac{\sum_{i=1}^{n} \frac{s_{im}\alpha_i^2}{w_i\alpha_i}}{\sum_{i=1}^{n} \frac{\alpha_i^2}{w_i\alpha_i}} = \frac{\sum_{i=1}^{n} \frac{s_{im}\alpha_i^2}{area(S_i)}}{\sum_{i=1}^{n} \frac{\alpha_i^2}{area(S_i)}} \quad (4)$$

where w_i is the support length of the fuzzy set. This is due to the fact that in the QM the certainties of a fuzzy set on the all levels of α-cut are equal. This fact indicates that our method is more general and reasonable than the QM.

Simulation: In our simulation, two membership functions corresponding to the NS and PS terms are in fat-shape form, and three membership functions corresponding to NZ, ZO, and PZ terms are in slim-shape form. We use inconsistent rules as shown in Table 1. The nonlinear plant model used in the simulation is given by a difference equation form, $y(k+1) = y(k)/(1 + y^2(k))u(k) + u(k)$. Using the inconsistent rules, simulation experiments have been carried out for two cases: Mamdani-QM, and Mamdani-

Table 1: Inconsistent rule table

e/ce	*nb*	*ns*	*nz*	*zo*	*pz*	*ps*	*pb*
nb	*nb*	*nb*	*nb*	*nb*	*ns*	*ns/nz*	*zo*
ns	*nb*	*nb*	*nb*	*ns*	*ns/nz*	*zo*	*pz/ps*
nz	*nb*	*nb*	*ns*	*ns/nz*	*zo*	*pz/ps*	*ps*
zo	*nb*	*ns*	*ns/nz*	*zo*	*pz/ps*	*ps*	*pb*
pz	*ns*	*ns/nz*	*zo*	*pz/ps*	*ps*	*pb*	*pb*
ps	*ns/nz*	*zo*	*pz/ps*	*ps*	*pb*	*pb*	*pb*
pb	*zo*	*pz/ps*	*ps*	*pb*	*pb*	*pb*	*pb*

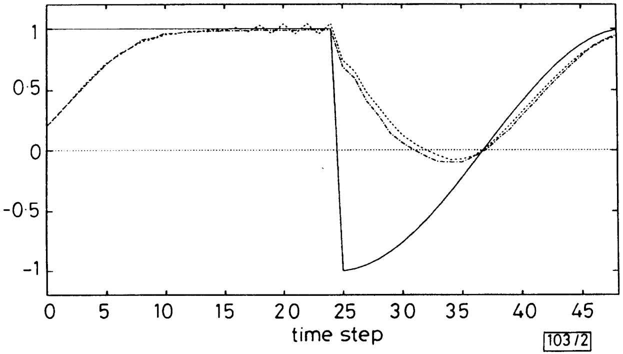

Fig. 2 Experiments of QM and LGM

——— Set points
· · · · · Mam-QM
– · – · Mam-LGM

LGM. The simulation result shows that the LGM defuzzification reduces the steady state error and settling time; and eliminates the chattering effect more successfully than the QM as shown in Fig. 2.

Conclusion: A new defuzzification method termed LGM was proposed for an FLC with multishaped or asymmetrical output fuzzy sets. An FLC using our defuzzification method was constructed and applied to the simulation of nonlinear plant control using typical inconsistent rules. Simulation results showed that our defuzzification method was more stable and robust than QM in the presence of inconsistent rules.

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