

## ANALYSIS OF THE EFFECT OF UT1-UTC TO HIGH PRECISION ORBIT PROPAGATION

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### ABSTRACT

As the spatial resolution of remote sensing satellites becomes higher, very accurate determination of the position of a LEO (Low Earth Orbit) satellite is demanding more than ever. Non-symmetric Earth gravity is the major perturbation force to LEO satellites. Since the orbit propagation is performed in the celestial frame while Earth gravity is defined in the terrestrial frame, it is required to convert the coordinates of the satellite from one to the other accurately. Unless the coordinate conversion between the two frames is performed accurately the orbit propagation calculates incorrect Earth gravitational force at a specific time instant, and hence, causes errors in orbit prediction. The coordinate conversion between the two frames involves precession, nutation, Earth rotation and polar motion. Among these factors, unpredictability and uncertainty of Earth rotation, called UT1-UTC, is the largest error source. In this paper, the effect of UT1-UTC on the accuracy of the LEO propagation is introduced, tested and analyzed. Considering the maximum unpredictability of UT1-UTC, 0.9 seconds, the meaningful order of non-spherical Earth harmonic functions is derived.

### 1. INTRODUCTION

As the technology for developing artificial satellites and Earth observation sensors has been progressed rapidly during the last decade, commercial Earth observation satellites providing 1m ground resolution images are expected to be launched in the very near future. The primary missions of high-resolution remote sensing satellites are high-precision topographic mapping and man-made target detection. In order to achieve these missions it is therefore essential to determine and predict accurate orbit of the satellite. Most of the high resolution remote sensing satellites have sun-synchronous and hence low Earth orbits (LEO). As well as the classical Keplerian force, LEO satellites experience four major perturbation forces which must be accurately modeled for precise orbit determination and propagation: non-spherical Earth gravity, third-body attraction, air drag and solar radiation pressure. Among these, the non-spherical Earth gravity is the most dominant perturbation factor for LEO satellites. The third-body attraction can effectively be confined to

lunisolar (Moon and Sun) attraction for LEO satellites and the ephemerides of Sun and Moon can be determined very accurately in the celestial frame. The most unpredictable terms are the spatial-temporal variations of solar radiation and atmospheric profile. The effective area of a satellite should also be determined for calculating precise effects of the air drag and the solar radiation pressure by considering external structure as well as time-varying attitude of the satellite.

The Earth gravity, major perturbation to LEO satellites, is related to the terrestrial reference frame not to the celestial frame. The Earth gravity model is represented by spherical harmonic functions with coefficients which have been determined by several international agencies (IAG 1971, NIMA 1997). Supposed that the Earth gravity model is perfectly determined, it is required to convert the coordinates of a satellite from the celestial frame to the terrestrial frame and vice versa. There are, however, two factors which are determined by periodic observation, measurement and short-term prediction for the celestial-terrestrial conversion. These are UT1-UTC and polar motion. The effect of the polar motion is however much smaller in comparison with that of UT1-UTC. The Advanced HPOP<sup>TM</sup> (Microcosm), which is one of the popular and accurate orbit prediction software and also included in STK<sup>TM</sup> does not take the effect of UT1-UTC into account. Unless the effects of UT1-UTC errors on the orbit propagation are not fully analyzed, the application of a large degree and order gravity model to the orbit propagation is not very meaningful. The application of a large degree and order gravity model requires a heavy computational cost (computational time is proportional to the square of the degree).

In this paper, we tested and analyzed the effect of UT1-UTC to the sun-synchronous orbit propagation. The results of the orbit propagation by using Earth gravity model with different orders are compared with the maximum propagation errors which can be caused by UT1-UTC. This gives an idea of the meaningful order of Earth gravity model when the UT1-UTC was not taken into account for the orbit propagator. Section 2 describes the definition of the terrestrial frame and Earth gravity model. The conversion of the celestial frame coordinates to the terrestrial frame including precession, nutation, Earth rotation and polar motion is described in Section 3. In Section 4, the time reference as well as the definition of UT1-UTC used in the orbit propagation is described. The experiments on the effect of UT1-UTC on the orbit propagation are shown in Section 5 and this paper is concluded in Section 6.

The main concern of this paper is the orbit propagation errors arising from not using UT1-UTC data for the Earth's spherical harmonic calculation. These errors may be comparable or even bigger than the errors caused by other improperly modelled perturbations such as air drag and radiation pressure.

## 2. TERRESTRIAL FRAME AND EARTH GRAVITY MODEL

The most widely used definition of the Earth terrestrial frame is a CTRS (Conventional Terrestrial Reference System) (Boucher 1990) which has the following criteria: 1) It is geocentric, the center of mass being defined for the whole Earth, including oceans and atmosphere, 2) Its scale is that of the local Earth frame, in the meaning of a relativistic theory of gravitation, 3) Its orientation was initially given by BIH (Bureau International de l'Heure) orientation of 1984.0, and 4) Its time evolution in orientation will create no residual global rotation with regards to the crust.

The CTRS which is monitored by IERS (International Earth Rotation Service) is called ITRS (International Terrestrial Reference System) (McCarthy 1996). The Z axis is the direction of the IRP (IERS Reference Pole) which corresponds to the direction of the BIH CTP (Conventional Terrestrial Pole) at epoch of 1984.0 with an uncertainty of 0.005" (seconds). The X axis is the intersection of the IRM (IERS Reference Meridian) and the plane passing through the origin and normal to the Z axis. The uncertainty of the IRM and the BIH zero meridian at the epoch of 1984.0 is also 0.005". These uncertainties of the definition gives less than 20cm for the positional error of LEO satellites. In order to realize this frame definition, IERS corrects various factors on each observation site such as solid Earth tide displacement, ocean loading, post glacial rebound, and atmospheric loading (McCarthy 1996). Earth gravity potential is defined by using spherical harmonic functions with respect to the CTRS mentioned above (NIMA 1997).

$$V = \frac{GM}{r} \left[ 1 + \sum_{n=2}^{n_{max}} \sum_{m=0}^n \left( \frac{a}{r} \right)^n P_{nm}(\sin \phi) (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) \right] \quad (1)$$

where GM is Earth gravitational constant, r is the distance from the Earth center of mass, a is the semi-major axis of Earth ellipsoid, n and m are degree and order,  $\phi$  and  $\lambda$  are geocentric latitude and longitude,  $C_{nm}$ ,  $S_{nm}$  are gravitational coefficients, and  $P_{nm}$  is associated Legendre function

The harmonic functions with 0 orders ( $m = 0$ ) are called zonal harmonics. Sectorial harmonics are the ones with  $n = m$  and the rest of the harmonic functions are called tesseral harmonics (Wertz 1978). The zonal harmonics depend only on the latitude and cause secular motion of satellites. The magnitudes of the tesseral and sectorial harmonics are smaller than those of the zonal harmonics. The two summation terms and the Legendre functions in eq. (1) give a steep increase of computational time by a small increase of the degree and order. Therefore, the degree and order of the gravity model to be used must be determined by considering the accuracy and the processing time.

Among the several gravity models which have been determined, published and updated, EGM96 (Earth Gravity Model 1996) of WGS84 (World Geodetic System) is considered as one of the most accurate and updated models currently (NIMA 1997). NIMA (National Imagery and Mapping Agency) in US which was formerly DMA (Defense Mapping Agency) has determined and published the WGS (World Geodetic System) series - WGS60, WGS 66, WGS72 and WGS84 by using global data from precise and accurate geodetic positioning, new observations of land gravity data, the availability of extensive altimetry data from GEOSAT, ERS-1 and TOPEX/POSEIDON satellites. EGM96 is the most improved model proposed by NIMA and it determines the GM, a and  $C_{nm}$ ,  $S_{nm}$  up to degree 360 in eq.(1).

The degree and order of the gravity harmonic function should be optimized for the accuracy required and computational time allowed for the satellite orbit propagation. Since the orbit propagation is performed normally in the celestial reference frame while the gravitational model is defined with respect to the terrestrial frame, it is necessary to convert the coordinates of a satellite between the two frames.

### 3. CELESTIAL FRAME AND COORDINATE CONVERSION

In 1991 the International Astronomical Union (IAU) decided that the IAU celestial reference

system would be realized by a celestial reference frame defined by the precise coordinates of extragalactic radio sources. The origin is to be at the barycenter of the solar system and the directions of the axes are to be fixed with respect to the quasars. The ICRS (IERS Celestial Reference System) was therefore defined by the J2000.0 equatorial coordinates of extragalactic objects determined from VLBI (Very Long Baseline Interferometry) (McCarthy 1996). The VLBI analyses provide corrections to the conventional IAU models for precession and nutation (Seidelmann 1982) and the accurate estimation of the shift of the mean pole at J2000.0 relative to its conventional one, to which the pole of the ICRS is attached.

The coordinate conversion between ITRS and ICRS can be performed by the classical procedure which makes use of the equinox for realizing the intermediate reference frame of data  $t$  (McCarthy 1996).

$$[\text{CRS}] = [\text{P}][\text{N}][\text{R}][\text{W}][\text{TRS}] \quad (2)$$

where  $[\text{P}]$  and  $[\text{N}]$  matrices correspond to the precession and nutation,  $[\text{R}]$  for Earth rotation and  $[\text{W}]$  for polar motion.

The polar motion is the movement of IRP axis due to internal motions and shape deformation of Earth. The dominant component of polar motion, called Chandler wobble, is a roughly circular motion of IRP around the celestial pole with an amplitude of about 0.7 arc seconds and a period of 14 months (Fisher 1999a). Shorter and longer time scale irregularities due to internal motions of Earth are not predictable and are monitored by IERS on weekly basis. Since the Chandler wobble can be modeled accurately, shorter term oscillation of IRP is very small (centimeter order) and the unpredictable long-term drift of IRP is also small compared with the time scale of tens of years, we can ignore the unpredictability of the polar motion in this study.

The precession and nutation terms are calculated as functions of either TDB (barycentric dynamic time) or TT (terrestrial dynamic time) (McCarthy 1996). The definition of different astronomical times is described in the next section. Although the time difference arises due to UTC (coordinated universal time) leap seconds (details in the next section), the several leap seconds give negligible effects on the long-term precession and nutation.

Finally, the Earth rotation factor is very sensitive to the difference between actual rotation time of Earth and universally fixed time. One second give rise to  $1/86400 \times 360 = 0.004$  arc degrees of Earth rotation and therefore approximately  $500 \text{ m}^2$  for the position of LEO satellites. This error results not only in the incorrect ground mapping of high resolution satellite images but also in incorrect calculation of Earth gravity acceleration. The effect of the cumulative Earth rotation rate errors to the orbit propagation accuracy is the issue of this paper. Since the errors in Earth rotation rate result only in longitudinal errors not in latitudinal errors, zonal harmonic functions are not affected. Only tesseral and sectorial harmonics with different orders are therefore concerned (see Section 2).

#### 4. ASTRONOMICAL TIMES AND UT1-UTC

There are two widely used time standards. One is the rotation of Earth and the other is the frequency of atomic oscillations (mainly the cesium-133 atom). Since the Earth's rotation is not uniform its rate exhibits both periodic changes and long term drifts on the order of a second per year.

Atomic standards are the closest approximations we currently have to a uniform time with accuracy on the order of microseconds per year (Seidelmann et al. 1992).

TAI (International Atomic Time) is the primary time standard in the world today and standard for the SI (System International) second. TT (Terrestrial Dynamic Time) is also based on TAI with a constant offset of 32.184 seconds in order to maintain the continuity of the pre-atomic time era. UT1 (Universal Time) is a measure of the actual rotation of Earth, independent of observing location. UT1 is essentially the same as the now discontinued GMT (Greenwich Mean Time). It is the observed rotation of Earth with respect to the mean sun corrected for the observer's longitude with respect to the Greenwich Meridian and for the observer's small shift in longitude due to polar motion. Since the Earth's rotation is not uniform, the rate of UT1 is not constant and its offset from atomic time is continually changing in a not completely predictable way. It is drifting 0.8 seconds per year currently with respect to the atomic time (Fisher 1999b).

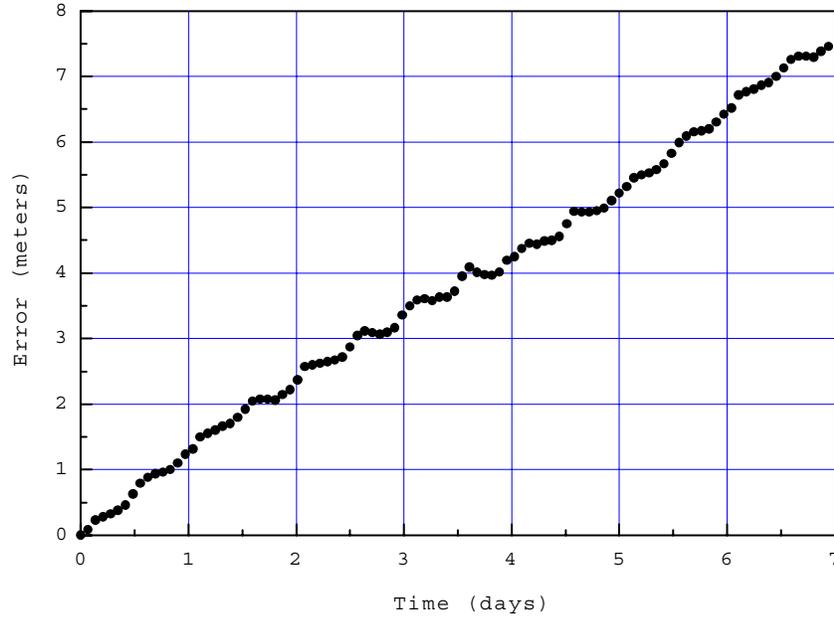
UTC (Coordinated Universal Time) which is currently broadcast worldwide and generally used for the satellite orbit propagation has the same rate of atomic time (TAI and TT). It, however, stays close to UT1 by adding integer numbers of seconds, called leap seconds, from time to time. This keeps solar noon at the same UTC averaged over the year, even though the rotation of Earth is slowing down. Up until 1999, 32 leap seconds have been added to UTC and their insertion is determined by IERS in order to keep the difference between UT1 and UTC (UT1-UTC) be less than 0.9 seconds. The UT1-UTC is measured by IERS on the weekly basis and can be accessed via internet.

This UT1-UTC difference causes significant error in ICRS-ITRS conversion. A maximum of 0.9 seconds give rise to errors in X, Y coordinates up to 450m. The incorrect coordinates in ITRS result in incorrect calculation of tesseral and sectorial terms in the gravity model and hence errors in the orbit propagation. Most of the high precision orbit propagation software are currently adopting very accurate gravitational model. The software which do not have capability of loading UT1-UTC measurements periodically degrade the propagation accuracy using the accurate gravity model. In this paper, the effects of a UT1-UTC error (maximum of 0.9 seconds) to the orbit propagation are analyzed.

## 5. EXPERIMENTS AND RESULTS

A high precision orbit propagation algorithm was developed for the current study. It used a fourth order Runge-Kutta integrator (Press et al. 1997) with the fixed time step of one second. The use of this short time step aimed at the accurate propagation ignoring computational time. EGM96 gravity model up to degree 60 was adopted to the algorithm. All other perturbation factors such as lunisolar attraction, air drag and solar radiation pressure were not included in order to see the gravitational effects only.

Firstly, the algorithm propagated a sample sun-synchronous ephemeris (epoch, position and velocity) for 7 days using 0 UT1-TUC and the full EGM96 degree 60 model. Then the same algorithm but using UT1-UTC of 0.9 seconds was applied. The difference of positional vectors (Euclidean distance) resulted from the two propagation outputs is plotted in Figure 1. Figure 1 shows the secular errors during the whole 7 days which were caused by incorrect tesseral and sectorial harmonic functions due to UT1-UTC.



**Figure 1.** Secular effect of UT1-UTC.

Figure 1 shows that the maximum 0.9 second error of UT1-UTC causes approximately 8m errors in 7-day orbit propagation with small and irregular short-term oscillations. The 1m/day secular effect of UT1-UTC may be smaller than or comparable to the other perturbation effects such as air drag and solar radiation pressure for LEO satellites depending on the size/mass ratio of the satellite.

The amount of this effect is then compared to that of tesseral and sectorial harmonic functions. Without applying any UT1-UTC error, the dependency of the orbit propagation errors on the tesseral and sectorial harmonics orders is calculated and plotted in Figure 2. The zonal harmonics with the full 60 degree were used in this case. As shown in Figure 2, the tesseral and sectorial harmonic functions with the orders larger than 40 result in less than 2m propagation errors during the whole 7 day period. The secular drift of 10m/7days occurred due to the tesseral and sectorial force of the order 35. Compared with the results shown in Figure 1, this result can lead to the conclusion that the use of the tesseral and sectorial harmonics with more than the order 40 is not very meaningful unless the UT1-UTC errors are taken into account.

Finally, the effects of the zonal harmonic degree to the orbit propagation errors are also tested and plotted in Figure 3. It also shows that the degree and order larger than 40 generates less than 3m error compared with the full 60 degree and order. Therefore, the application of zonal, tesseral and sectorial harmonics with a degree and order larger than 40 without applying UT1-UTC is not meaningful.

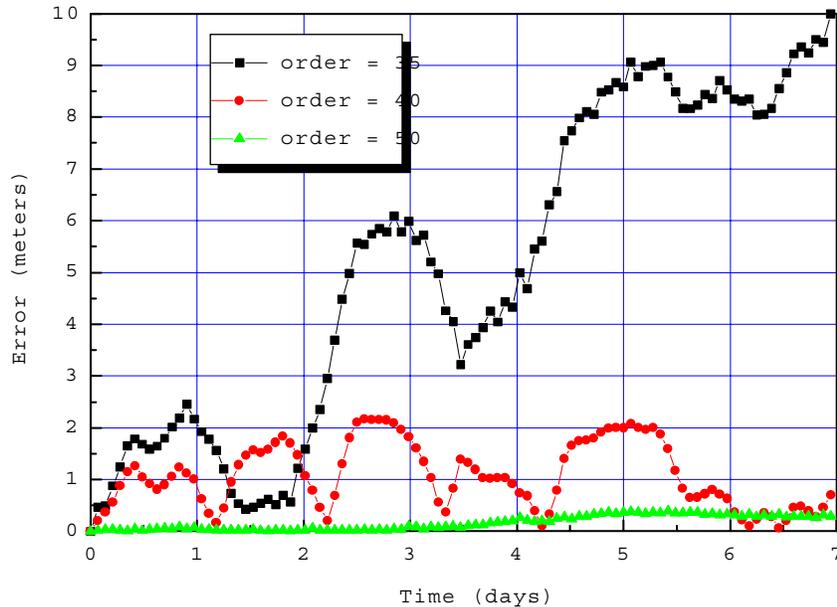


Figure 2. Errors of tesseral and sectorial harmonics with different orders.

## 6. CONCLUSION

In this paper, the effects of unpredictable Earth rotation rate, UT1-UTC, on the orbit propagation are analyzed. The UT1-UTC error of a maximum 0.9 seconds can cause approximately 1m/day secular orbit propagation error with small short-term oscillations. This is due to the application of incorrect tesseral and sectorial gravity harmonic functions to the orbit propagation. It was also shown that the use of gravity harmonic functions with degrees and orders larger than 40 is not very meaningful unless correct UT1-UTC data is continually updated to the orbit propagation algorithm. This concludes that UT1-UTC values must be inserted to the orbit propagator for the accurate orbit prediction rather than applying higher degree and order gravity functions which result in serious increase in processing time.

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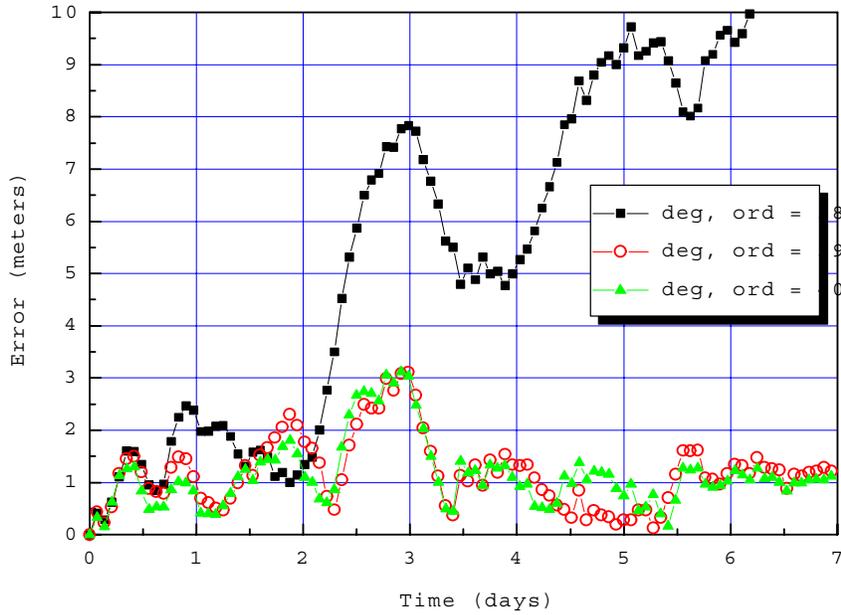


Figure 3. Errors caused by all harmonics.

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