Representing Continuous Processes in Discrete Events: A Combined Modeling Approach

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ABSTRACT

Combined models, specified by two or more modeling formalisms, can represent a wide variety of complex systems. This paper describes a methodology for the development of combined models in two model types of discrete event and continuous process. The methodology is based on transformation of continuous state space into discrete one to homomorphically represent dynamics of continuous processes in discrete events. As an example, a combined model of human heart is developed which incorporates conventional differential equation formalism with Zeigler's DEVS (Discrete Event Systems Specification) formalism.

1. INTRODUCTION

Modeling simulation of human's heart has been employed as an effective tool to test hypotheses on circulation physiology and pathology. In the past, several electric analog models for heart have been proposed [Pater and van den Berg, 1964][Beneken and De Wit, 1967]. Such models are represented by a collection of R-L-C (Resistors-Inductors-Capacitors) components and some forcing functions connected together. Since the models are so complex, simulation usually employs facilities to reduce programming efforts such as digital simulation languages including ACSL and CSSL [Korn and Wait, 1978].

Combined models are represented by two or more model types out of four possible types in the cross product of time domain and state space [Fishwick and Zeigler, 1991]. One such model is combined discrete event and continuous model. Recently, a system-theoretic framework for constructing formal models of combined discrete event with continuous process has been proposed by [Zeigler, 1989]. The framework employs Zeigler's DEVS (Discrete Event Systems Specification) formalism to homomorphically represent dynamics of a continuous process by means of discrete events. As recognized in [Fishwick and Zeigler, 1991], the framework is more adequate for representing complex systems containing phase transitions and complicated boundary conditions than the original work in system dynamics [Shearer, Murphy, and Richardson, 1967].

This paper describes a methodology for the development of a combined model of the human heart system. The methodology is based on transformation of continuous state space into discrete one to represent dynamics of continuous processes in discrete events proposed by [Zeigler, 1989]. However, we propose a formal structure to represent combined discrete event and continuous processes at the atomic model level. Similar to those taken in [Praehofer, 1991] and [Fishwick and Zeigler, 1991] the structure is combination of the atomic DEVS structure and additional information which accounts for dynamics of continuous models. More specifically, within the structure, modeler needs to specify a continuous sub-model specification for each set of contiguous states called a phase.

Rest of this paper is organized as follows. Section 2 briefly reviews the DEVS formalism. Section 3 describes the theory of representation of a continuous model in a discrete event model within the DEVS formalism. Section 4 presents a formal definition for combined discrete event and continuous models. Heart is described in brief in section 5 and its combined model is presented in section 6. Section 7 concludes the paper.

2. THE DEVS FORMALISM: A SYSTEM-THEORETIC APPROACH

A set-theoretic formalism, the DEVS formalism [Zeigler 1984], specifies discrete event models in a hierarchical, modular form. Within the formalism, one must specify 1) the basic models from which larger ones are built, and 2) how these models are connected together in hierarchical fashion. A basic model, called an *atomic model* (or *atomic DEVS*), has specification for dynamics of the model. An atomic model M is specified as:

$$M = < X, S, Y, \delta_{int}, \delta_{ext}, \lambda, ta >$$

X : input events set;
S : sequential states set;
Y: output events set;
$\delta_{int} : S \rightarrow S$: internal transition function;
$\delta_{ext} : Q \times X \rightarrow S$: external transition function;
$Q = \{ (s, e) \mid s \in S, 0 \leq e \leq ta(s) \}$: total state of M;
$\lambda : S \rightarrow Y$: output function;
ta: $S \rightarrow$ Real : time advanced function.

The second form of the model, called a *coupled model* (or *coupled DEVS*), tells how to couple (connect) several component models together to form a new model. This latter model can itself be employed as a component in a larger coupled model,thereby giving rise to construction of complex models in hierarchical fashion. A coupled model DN is defined as:

$$DN = < D, \{M_i\}, \{I_i\}, \{Z_{i,j}\}, SELECT >$$

D : component names set;
for each i in D,
  M_i : DEVS for component i in D;
  I_i : set of influencees of i ;
for each j in I_i,
  $Z_{i,j} : Y_i \rightarrow X_j$: i-to-j output translation function;
SELECT: subsets of $D \rightarrow D$: tie-breaking selector.

Detail descriptions for the definitions of the atomic and coupled DEVS can be found in [Zeigler 1984].

3. REPRESENTING CONTINUOUS MODELS IN DEVS

The concept of discrete events arised out of simplification of real-world complexity through classification that takes dynamics into account. Recently, Zeigler introduced an approach to representing a deterministic continuous system in a discrete event model[Zeigler, 1989]. Originally introduced for the event-based control paradigm, the representation makes it possible to combine discrete event models with continuous model components [Fishwick and Zeigler, 1991]. Mapping a continuous system into a discrete event system is based on the following assumptions.

A-1) The input stimulations of the original system are piecewise constant time functions.

A-2) The output is a finite set.

A-3) The state space can be partitioned into a finite set of mutually exclusive blocks, each being equivalent class of states with respect to the output.

With the assumptions above, [Zeigler, 1989] showed sufficient conditions for a homomorphic relation between the original system and its discrete events representation at the input-output trajectories level. Note, however, that a discrete event model so represented doesn't necessarily preserve the internal structure of the original. The homomorphism discussed above ensures that only at the time points at which the original output changes values, the corresponding DEVS model outputs the new value equivalent to the original output value.

We briefly describe how to constitute a DEVS model to represent a continuous system based on the assumptions above. Let a continuous system be M_c and a corresponding atomic DEVS be M_{devs}. By the assumption A-1), the input of M_c is a sequence of step functions. The assumptions A-2) and A-3) make it possible to discretize M_c's state space that can be mapped into the state space of M_{devs}. To be specific, the state space of M_{devs} is a collection of discrete values that are boundaries of mutually exclusive blocks of equivalent classes of states in M_c.

To construct such a M_{devs} using the methodology introduced in [Zeigler, 1989], we need to specify four characteristic functions within the atomic DEVS structure described in section 2. The functions are the internal transition function, the external transition function, the time advanced function, and the output function. In [Zeigler, 1989], four approaches to obtain characteristic functions were proposed. Here we briefly introduce two approaches.

(Case 1) If the continuous system is modeled by an analytically tractable differential equations, then the characteristic functions can be expressed in closed form solutions.

(Case 2) If the continuous system is modeled by differential equations which can be numerically solved in advance, then the characteristic functions are obtained in form of tabular approximations by simulating the system.

The approaches above provides a mathematical start toward understanding the nature of discrete event concepts. However, the representation method requires us to identify events that partitions the output set of the original system into the equivalent blocks described above. Although no formal method has been known for such identification, some heuristic suggested in [Fishwick and Zeigler, 1991] are as follows.

- A significant change in input, output, or state trajectories can be an event.
- Interaction with objects to be modeled, such as departure and arrival, are possible events.
- Separated sub-models can be seen as phases between which model context switches can occur. Such phase changes can be events.

4. COMBINED MODEL DEFINITION

Although combined models are represented by two or more model types, our concern in this paper is a combined discrete event/continuous model. Our objective is to define a formal structure to represent continuous systems within the DEVS formalism. The definition shall support specification of combined discrete event/continuous models in a manner similar to the DEVS formalism. Following the formalism, we define two classes of combined models: atomic and coupled combined model. An atomic combined model has specification of phase transition while a coupled combined model has component combined models and their coupling specification. An atomic combined model is defined as a structure:

$$M_{com} = < X, S, Y, \delta_{int}, \delta_{ext}, \lambda, ta, f, CM >$$

f : set of phases of contiguous states → CM;
CM : set of continuous models;
X, S, Y, δ_{int}, δ_{ext}, λ, and ta are the same as the atomic DEVS in section 2.

Note that continuous models in the CM can typically be differential equations models. Definition of a coupled combined model remains unchanged.

5. HEART SYSTEM IN BRIEF

Human's heart can be viewed as a pump. A hollow muscular organ, the heart contains four chambers, namely left atrium, left ventricle, right atrium, and right ventricle. Let us briefly explain the heart cycle and associated events. Heart rate for a normal adult in rest is approximately 72 /sec, equivalently 0.7 to 0.8 sec/cycle. *Systole* (lasting about 0.3 sec) and *diastole* (lasting about 0.5 sec) are two main parts of the heart cycle. Systole is the active part of the heart cycle. It comprises two phases: isovolumic contraction phase (ICP) (lasting about 0.06 sec) and ejection phase (EP) (lasting about 0.28 sec). Diastole comprises three phases: isovolumic relaxation phase (IRP) (lasting about 0.06 sec), filling phase (FP) (lasting about 0.3 sec), and atrial systole phase (ASP) (lasting about 0.16 sec).

We now discuss changes in three pressures, namely aortic pressure, ventricular pressure, and atrial pressure for each phase during one cardiac cycle discussed earlier. During ICP, the ventricular pressure rises rapidly until the aortic valve opens which enters EP. During the same phase, the aortic pressure is linearly decreasing while the atrial pressure is linearly increasing. EP comprises two sub-phases: rapid and reduced phase. The rapid phase(EP1) takes the first third of EP during which blood begins to pour out of the ventricles, with about 70 % of the emptying. The reduced phase(EP2) lasts for the next two thirds of EP during which the remaining 30 % is to be empty. During the rapid phase(EP1), both aortic and ventricular pressures are parabolically increased while atrial pressure is less rapidly decreased. In the reduced phase(EP2), atrial pressure is linearly increasing while the other two are parabolically decreasing. In IRP with aortic valves closed, both aortic and atrial pressures are increasing sightly; ventricular pressure is rapidly decreasing. In FP, aortic pressure is linearly decreasing while both ventricular and atrial pressures are parabolically decreasing for the about one fifth of FP, then increasing linearly for the remaining period. During FP, owing to the closure of the atrioventricular valves large amounts of blood is accumulated in the atria.

The rapid phase(FP1) takes the first third of FP during which blood flows into the ventrioles; this is blood that continues to empty into the atria from the veins and passes on through the atria directly into the ventrioles.

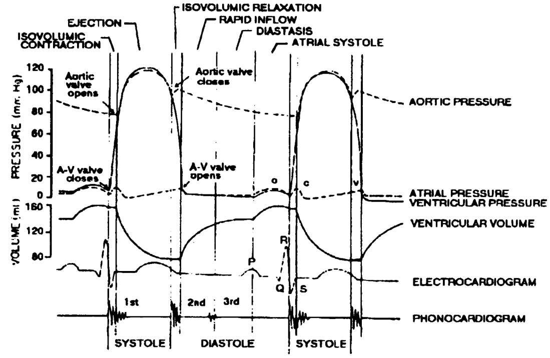


Fig. 1. Changes in Pressures in a Heart Cycle.

The reduced phase(FP2) lasts for the next two thirds of FP, when the inflow of blood into the ventrioles is almost at a standstill, is called diastasis.

Finally, in ASP, the atria contract and give an additional thrust to the inflow of blood into the ventriole; this accounts for approximately 20 to 30 percent of the filling of the ventrioles during each cardiac cycle.

In the phase(ASP), aortic pressure is linearly decreasing. On the other hand, ventricular is slowly increasing and atrial pressure is slowly increasing and then decreasing.

Fig. 1 shows changes in pressures during the cardiac cycle.

6. COMBINED MODEL OF HEART SYSTEM

We are now ready to develop a combined model of heart system based on a formal structure of M_{com} defined in section 4 . First of all, the following assumptions are made for correctness.

(1) Heart system meets assumptions A-1), A-2), and A-3) for input, output, and state set described in section 3.

(2) Heart system is modeled by a set of differential equations.

We shall take the following steps to complete a combined model. First, we identify phases in a heart cycle which are basis mapping continuous state space to discrete one. Next, we constitutes a set CM (Continuous Model) of submodels within M_{com} described in section 6 by developing a continuous model of analog circuits for each phase. This is followed by defining the mapping function f of M_{com} and discrete events portioning. Finally, obtaining characteristic functions of the heart system completes M_{com}.

<u>Phase Identification and Partioning</u>

With the assumptions above, we employ the third heuristic for identifying events in the continuous process in which phase transitions are activated by a list of events. Employing such heuristic is because phase changes in cardiac cycle are compatible with those occurred in the discrete event world. Fig. 2 shows phase partitioning, phase transitions, and state transitions in a heart cycle. With the phases defined above, we need to develop a set of continuous models, each of which is identified with a phase.

Electric Sub-Models

Generally, blood flow in a human body involves with a sequence of two circulations: systemic and pulmonary. In the systemic circulation, blood enter into the left atrium through the pulmonary vein after exchanging gas in the lung. Then it enters into the left ventricle through mitral valve. After then, blood flows out through the aortic valve during the left ventricular contraction, eventually returns into the right atrium through artery, arteriole, capillary, venula, and vein. The pulmonary circulation has similar circulation paths to those of the systemic circulation.

A cardiac cycle consists of a period of relaxation called diastole followed by a period of contraction called systole. A cardiac cycle has seven phases: atrial systole phase (ASP), isovolumic contraction phase (ICP), rapid ejection phase (EP1), reduced ejection phase(EP2), isovolumic relaxation phase (IRP), rapid filling phase (FP1), and reduced filling phase(FP2). Note that the changes in the phases are caused by events. We now develop electric models of heart for the seven phases which constitute the set of differential equation models in a combined model of the heart system. Since electric models in pulmonary circulation are similar to those in systemic circulation, models only in the systemic circulation are presented.

In ASP, blood normally flows continually from the great veins into the atria. Approximately 70 percent of the flow goes through the atria into the ventricles even before the atria contracts. Then, atrial contraction usually causes additional 20 to 30 percent filling of the ventricles. Therefore, the atria simply function as primer pumps which increase the ventricular pumping effectiveness as much as 30 percent. The heart system in the systemic circulation for ASP can be modeled as an electric circuit of Fig.3 a. Note that we assume the atria of heart to be passive compliances, thereby being modeled by a constant capacitor.

In ICP, the ventricular pressure abruptly rises causing A-V valves (atrioventricular) to close. Then an additional time is required for the ventricle to build up sufficient pressure to push the semilunar (aortic and pulmonary) valves open against the pressures in the aorta and pulmonary artery. Therefore, during this period, contraction is occurring in the ventricles, but there is no emptying. The heart system in the systemic circulation for ICP can be modeled as an electric circuit of Fig.3 b. Similarly, the heart system in the systemic circulation for the phases EP1, EP2, IRP, FP1, and FP2 can be models as electric circuits of Fig. 3c, 3d, and 3e, respectively.

Other steps

With the electric sub-models in Fig. 3 and associated phases, we partition discrete event of heart system as shown in Table. 1. With information in the table, we complete a combined model of the heart system shown in Fig. 4.

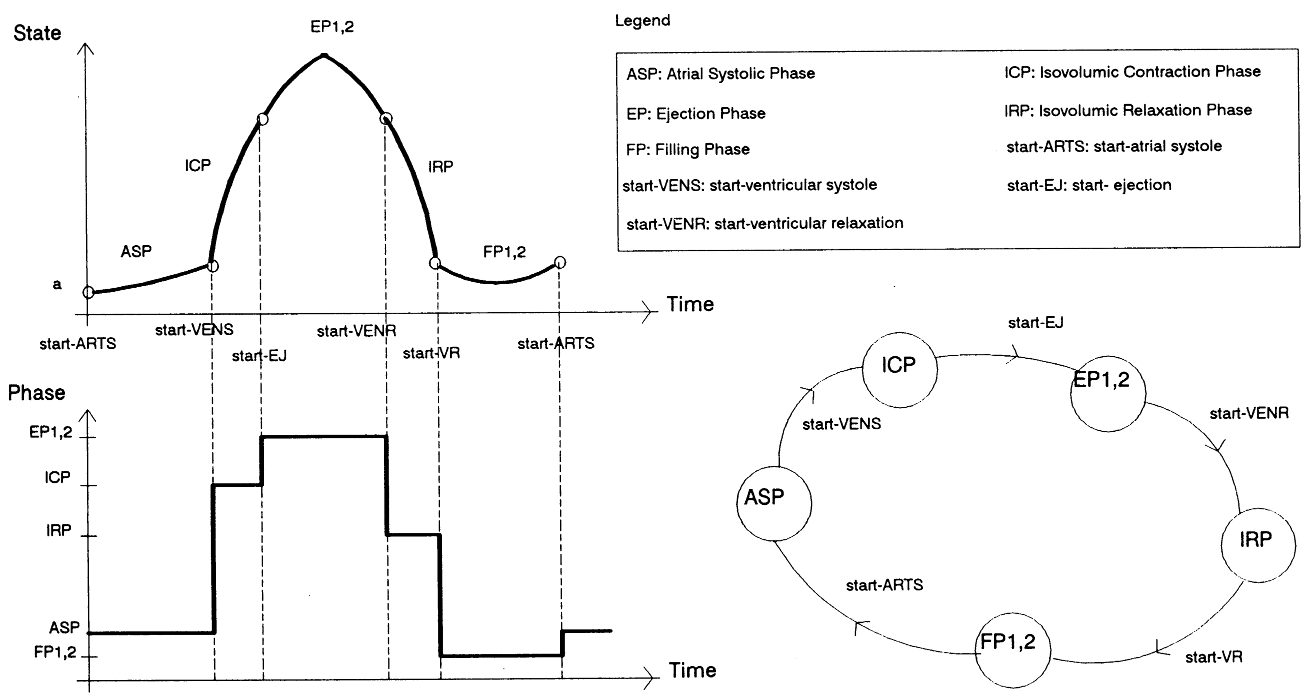
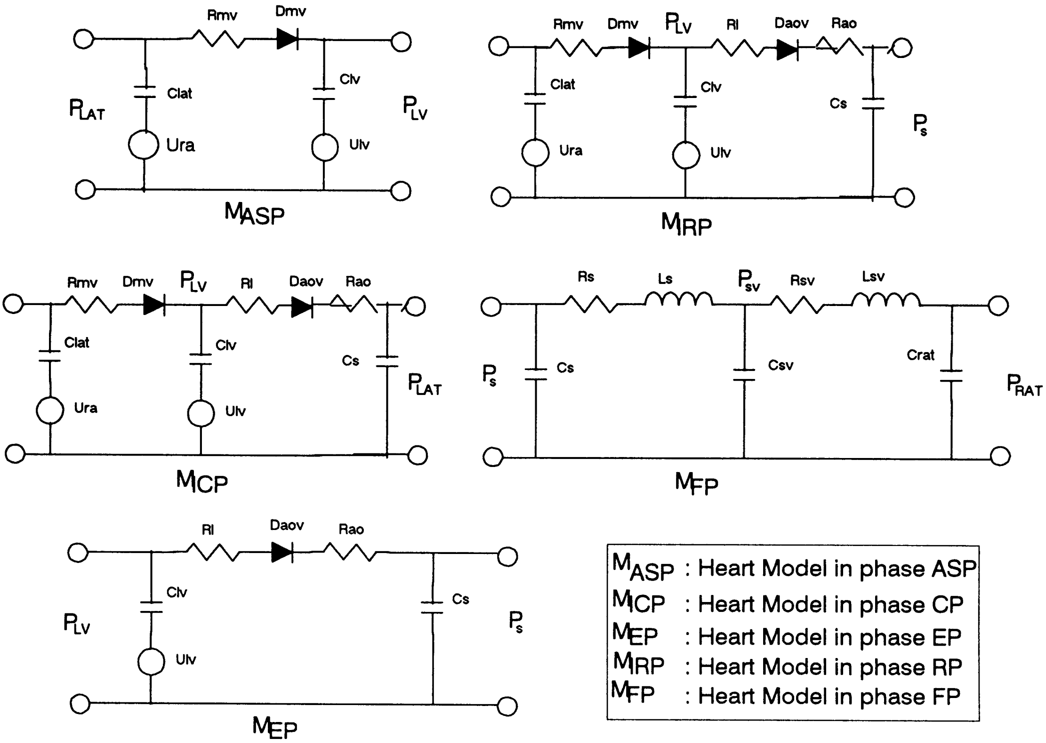


Fig. 2. Phase Partitioning and Transitions.



	Systemic Circulation		Pulmonary Circulation	
State	P_{lat}	: 11.294	P_{rat}	: 3.3285
	P_{clv}	: 112.76	C_{rv}	: 105.52
	P_s	: 71.112	P_{pa}	: 13.417
	P_{sv}	: 70.516	P_{pr}	: 13.393
	Q_s	: 8.8880	Q_p	: 0.78636
	Q_{sv}	: 67.337	Q_{pp}	: 23.836
Parameter	U_{lv}	: 60	U_{rv}	: 25
	E_{ld}	: 01.	E_{rd}	: 3.0E-02
	E_{ls}	: 1.3750	E_{rs}	: 0.3288
	R_l	: 8.0E-02	R_r	: 1.750E-02
	R_{ao}	: 3.751E-03	R_{tm}	: 3.751E-03
	R_{mv}	: 3.752E-03	R_{pr}	: 3.751E-03
	C_s	: 0.22	C_{rat}	: 20
	R_s	: 6.75E-02	C_{pal}	: 9.0E-02
	L_s	: 8.25E-04	R_p	: 3.376E-02
	C_{sv}	: 1.46	L_p	: 7.50E-04
	R_{sv}	: 1.0	C_{pv}	: 2.67
	L_{sv}	: 3.60E-03	R_{pp}	: 0.1013
	C_{lat}	: 46.7	L_{pp}	: 3.08E-03

Fig.3. Sub-models of Heart System in Systemic Circulation and Parameters.

7. CONCLUDING REMARKS

Combined models can support specification of a wider variety of complex systems which are to be defined using many different modeling formalisms. We have described a modeling methodology which combines discrete event models with continuous ones in one framework. The methodology has been based on transformation of continuous state space into discrete one to homomorphically represent dynamics of continuous processes in discrete events. Simulation results for the modeled heart system is shown in Fig. 5.

A formal structure to combine differential equations formalism and Zeigler's DEVS formalism has been presented. Based on the DEVS formalism, the structure has included additional information on continuous models. Combination has been based on mapping of differential equation sub-models in phases of DEVS models at the atomic level.

A combined model of human heart has been developed to demonstrate the methodology. In modeling, for each phase of heart cycle, we have developed sub-models of heart system in continuous processes and assigned them to associated phases of a discrete event system. Simulation of a combined model of the heart system developed here needs an environment which supports discrete event modeling simulation within the DEVS formalism. One such environment can be DEVSIM++ [Kim and Park, 1992] which realizes the DEVS formalism in C++.

The results indicate that the proposed framework employs adequate for representing complex heart system contain phase transition and complicated boundary conditions than the original work in the heart system.

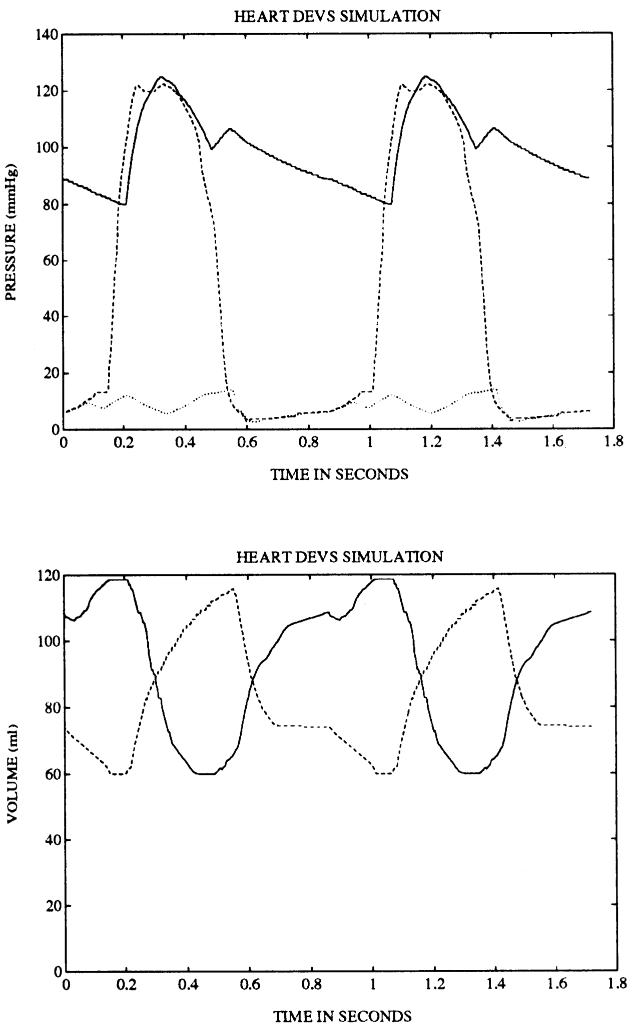


Fig. 5. Simulation Results for the Modeled Heart System.

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