Effectiveness Analysis Model

MOP
Speed,
Operation Time,
…

X_i

Effectiveness
Analysis Model
$f(X)$
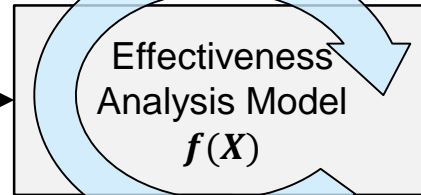
n-th simulation evaluations

MOE
Success(1)/Fail(0),
Survive(1)/Destroy(0),
…

$f(X_i) = Y_i \sim Bernoulli(p_i)$

$$p_i = \lim_{n \to \infty} \frac{\sum_{k=1}^{n} Y_i}{n}$$

Find p_i(Measure of Effectiveness, MOE) from given X_i(Measure of Performance, MOP)

Simulation Based Optimization (1/2)

- For Effectiveness Analysis Model
  - Find input scenarios that maximizes(or minimizes) p_i

$f(X)$: Effectiveness Analysis Model
p_i: Effectiveness of X_i decided by $f(X)$
$\boldsymbol{X}_o$: Optimal Solution Set
$X_i = [x_1, x_2, \ldots, x_n]$: Input Scenario
Θ: Range of input scenarios
n: # of simulation evaluations

$$\boldsymbol{X}_o = \arg \max_{X \in \Theta} p_i = \arg \max_{X \in \Theta} \left[\lim_{n \to \infty} \sum_{k=1}^{n} f(X_i)/n \right]$$

  - In real world, infinite evaluations is impossible.
    ➔ Get point estimate $\widehat{p}_i$ by limited n-th evaluation

$$\widehat{p}_i = p_i + N(0, p_i(1 - p_i)/n) = p_i + \boldsymbol{noise}$$

How can we find input scenarios that maximizes(or minimizes) $\boldsymbol{p_i}$ with reducing an effect of the $noise$ in limited evaluations?

Simulation Based Optimization (2/2)
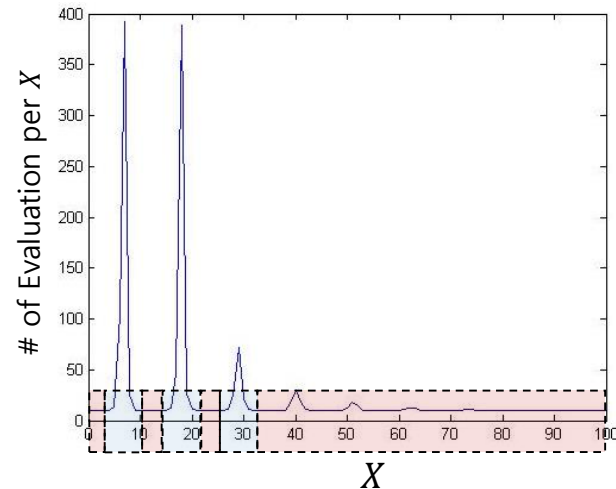
- To minimize an effect of the $noise$
  - Increase n (# of evaluations)
    - Increasing n for all input scenarios is inefficient
  - Increase n for input scenarios which has a probability to be the optimal solution and decrease n for the others
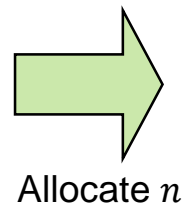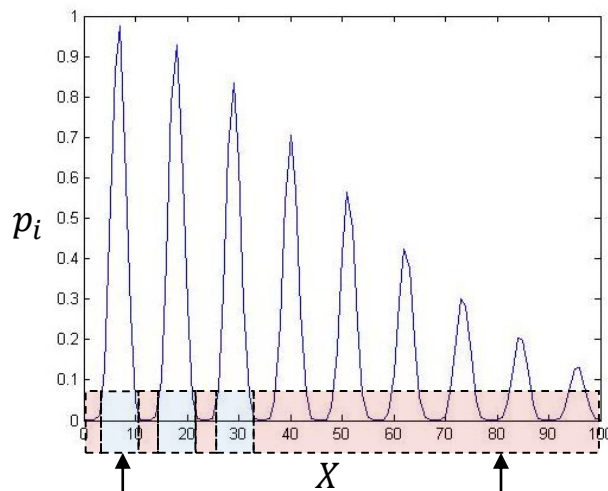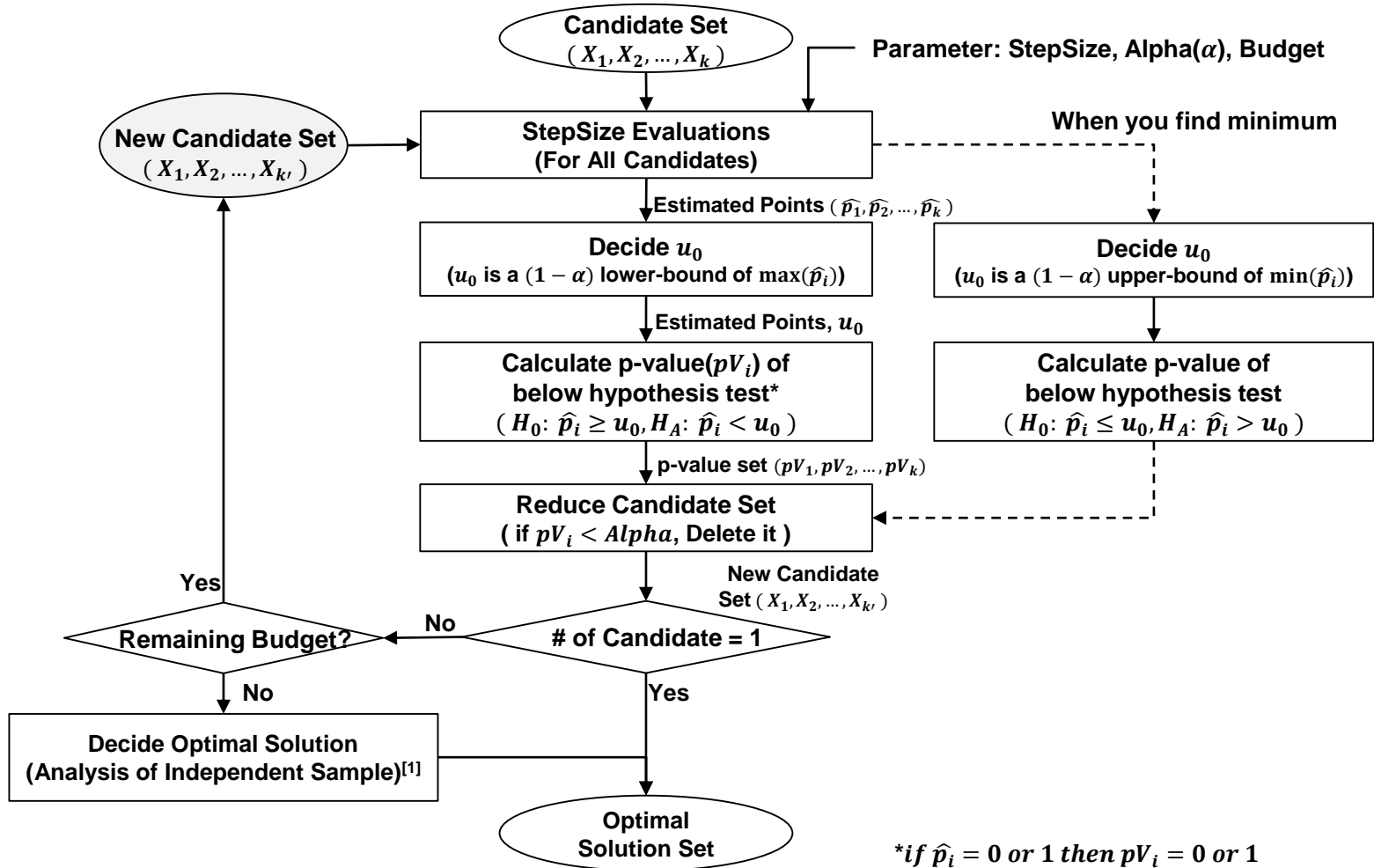

Allocate n

The others ➜ Decrease n

More likely to be the optimal solution ➜ Increase n

Proposed Algorithm

Proposed Algorithm (1/2)

Candidate Set $(X_1, X_2, \ldots, X_k)$

Parameter: StepSize, Alpha(α), Budget

New Candidate Set $(X_1, X_2, \ldots, X_{k'})$

StepSize Evaluations (For All Candidates)

When you find minimum

Estimated Points $(\widehat{p_1}, \widehat{p_2}, \ldots, \widehat{p_k})$

Decide u_0
(u_0 is a $(1 - \alpha)$ lower-bound of $\max(\widehat{p_i})$)

Decide u_0
(u_0 is a $(1 - \alpha)$ upper-bound of $\min(\widehat{p_i})$)

Estimated Points, u_0

Calculate p-value(pV_i) of below hypothesis test*
(H_0: $\widehat{p_i} \geq u_0$, H_A: $\widehat{p_i} < u_0$)

Calculate p-value of below hypothesis test
(H_0: $\widehat{p_i} \leq u_0$, H_A: $\widehat{p_i} > u_0$)

p-value set $(pV_1, pV_2, \ldots, pV_k)$

Reduce Candidate Set
(if $pV_i < Alpha$, Delete it)

New Candidate Set $(X_1, X_2, \ldots, X_{k'})$

Yes

No

Remaining Budget?

# of Candidate = 1

No

Yes

Decide Optimal Solution (Analysis of Independent Sample)[1]

Optimal Solution Set

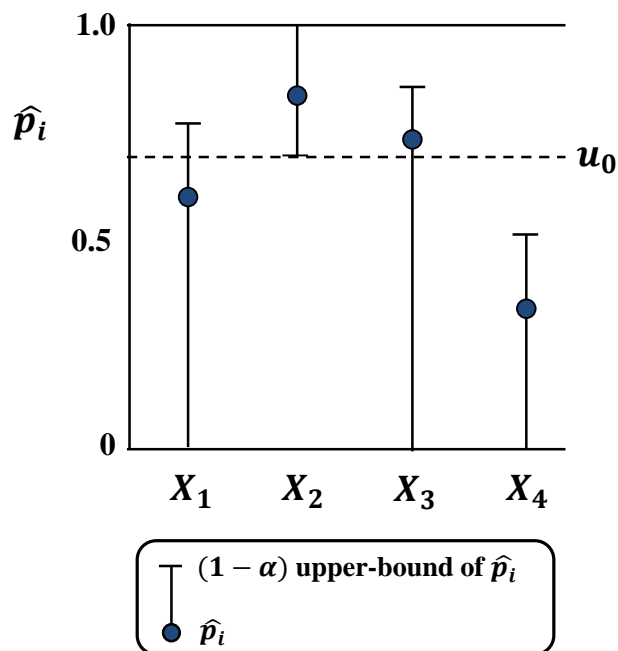*if $\widehat{p_i} = 0$ *or* 1 *then* $pV_i = 0$ *or* 1

[1] Walpole, Ronald E., et al. *Probability and statistics for engineers and scientists*. Vol. 5. New York: Macmillan, 1993.
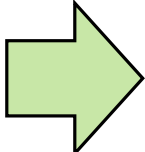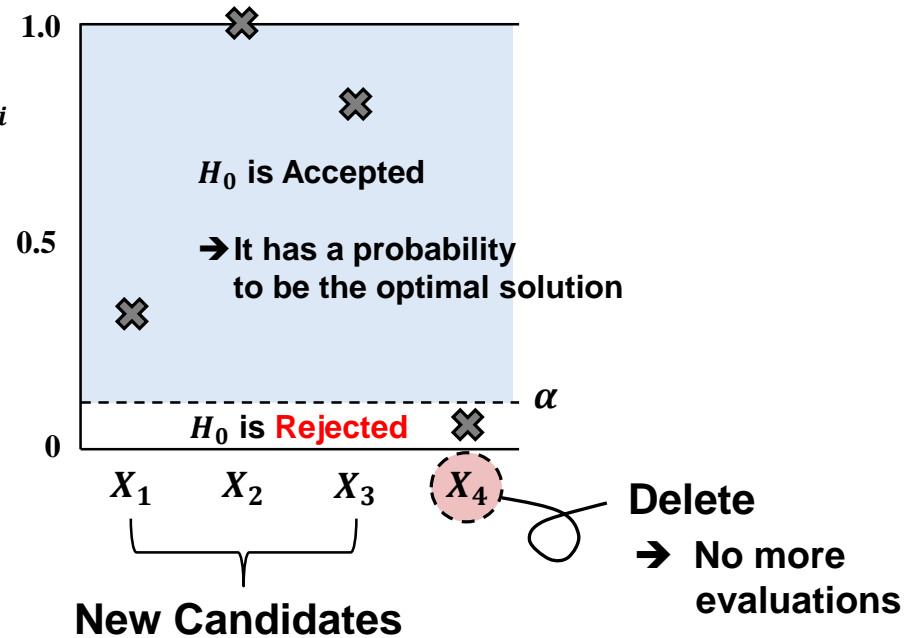
Proposed Algorithm (2/2)

- Hypothesis Test[1]
  - $H_0: \hat{p}_i \geq u_0, H_A: \hat{p}_i < u_0$
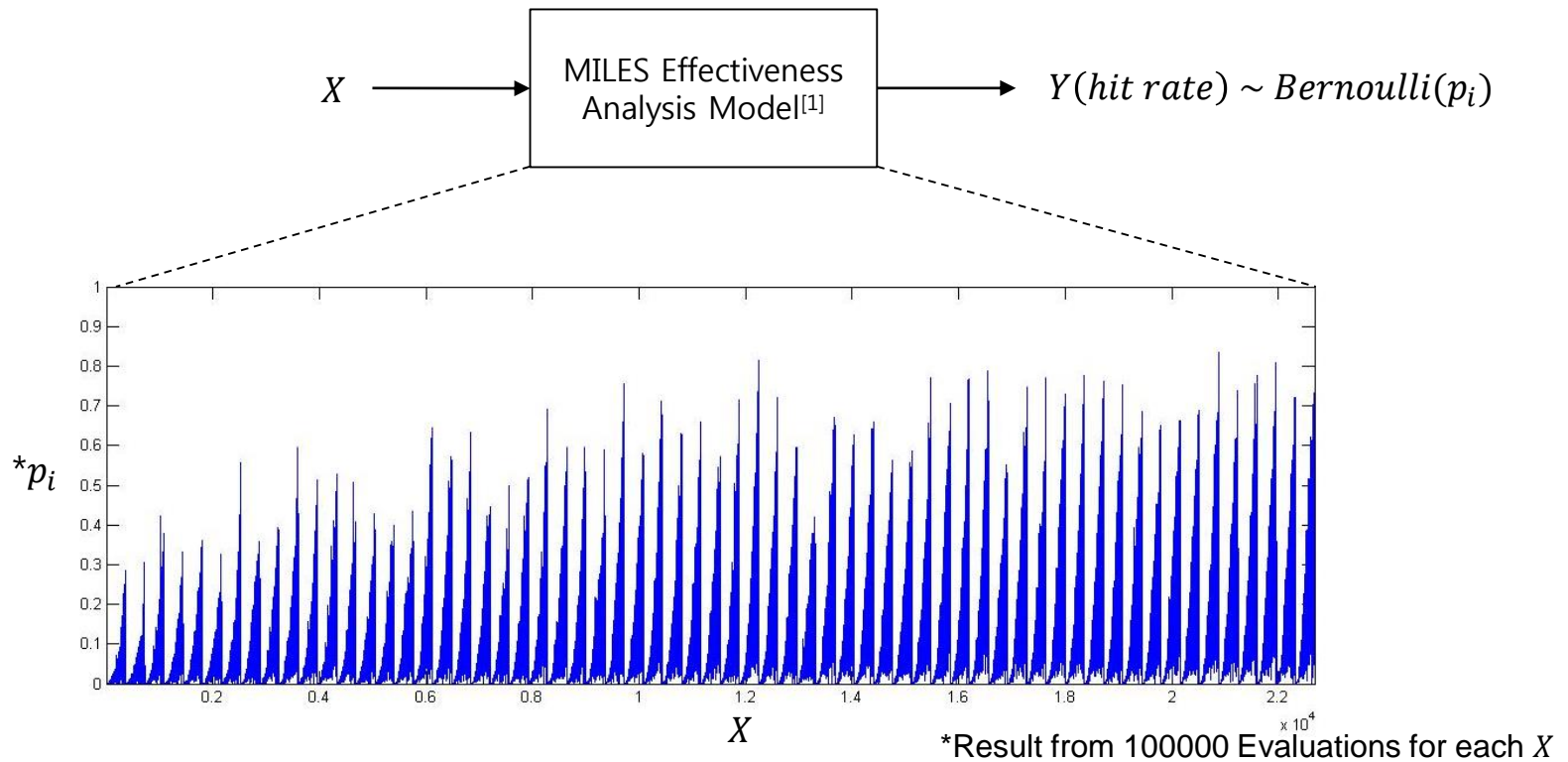 - $u_0 = \max(\hat{p}_i) - t_{\alpha,n-1} \times \sqrt{\max(\hat{p}_i)(1 - \max(\hat{p}_i))/n}$

[1] Walpole, Ronald E., et al. *Probability and statistics for engineers and scientists.* Vol. 5. New York: Macmillan, 1993.
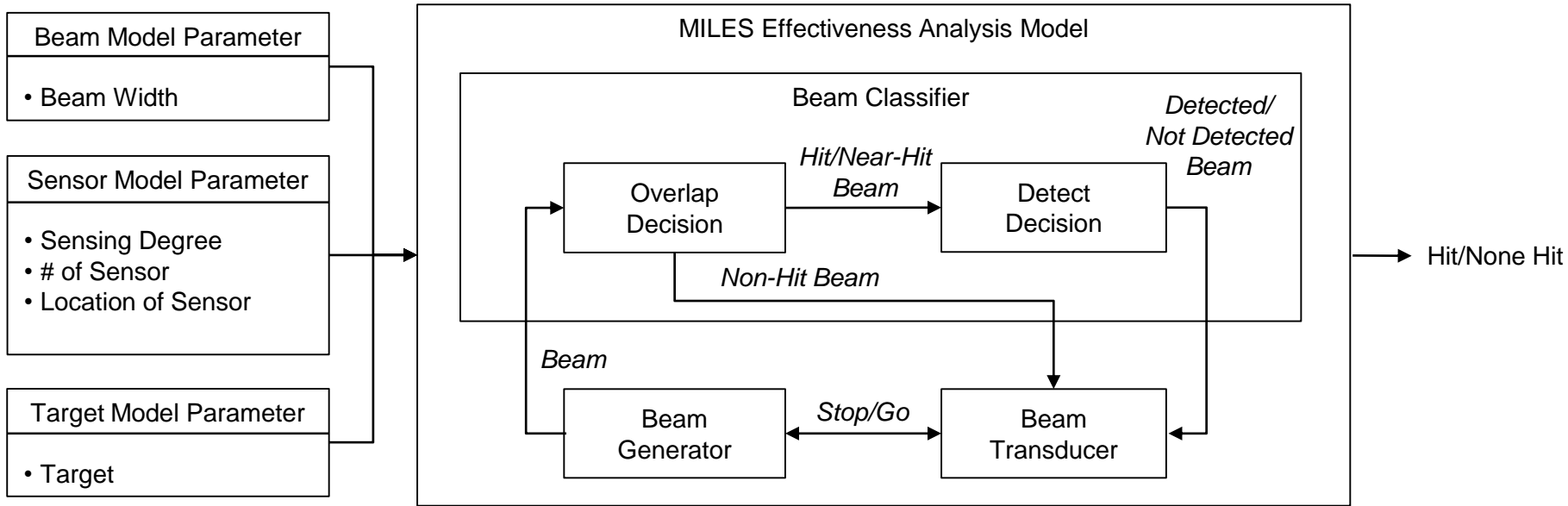
Case Study: MILES Model (1/2)

- Multiple Integrated Laser Engagement System (MILES)
    - Find input scenarios to maximize effectiveness(hit rate)
    - Input scenario = [Beam Width, Sensing Degree, # of Sensor, Location of Sensor, Target]

$$X \longrightarrow \boxed{\text{MILES Effectiveness Analysis Model}^{[1]}} \longrightarrow Y(hit\ rate) \sim Bernoulli(p_i)$$

$*p_i$

X

*Result from 100000 Evaluations for each X

[1] 김탁곤, 최선한, 이순주, 최창범, 박판준, 최태영, 김수범. "KCTC 마일즈 장비의 명중 감지율 계산을 위한 광 공학 모델 개발 및 활용 방안," 제 5회 육군 M&S 학술 대회, 2012년 11월

Case Study: MILES Model (2/2)

Beam Model Parameter

• Beam Width

Sensor Model Parameter

• Sensing Degree
• # of Sensor
• Location of Sensor

Target Model Parameter

• Target

MILES Effectiveness Analysis Model

Beam Classifier

Detected/ Not Detected Beam

Overlap Decision → *Hit/Near-Hit Beam* → Detect Decision

Overlap Decision → *Non-Hit Beam*

Beam

Beam Generator ←→ *Stop/Go* ←→ Beam Transducer

→ Hit/None Hit

Parameter	Range
Beam Width (cm)	{ 5, 10, …, 100 } (20)
Sensing Degree (°)	{ 5, 10, …, 90} (18)
# and location of Sensor	{ P_1, P_2, …, P_{63} } (63)
Target	{ T_1 } (1)

# of Input
$20 \times 18 \times 63 \times 1 = 22680$

✕

Time for 1 Evaluation
0.000524 sec

✕

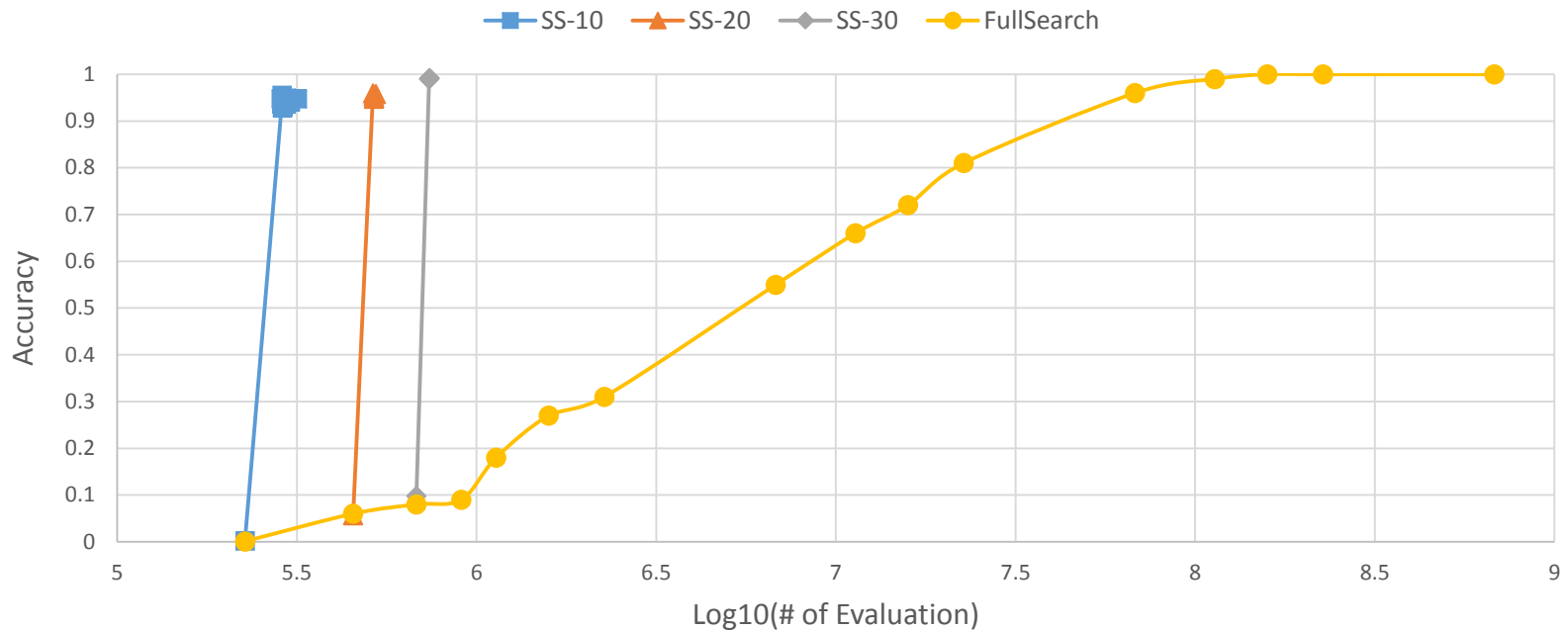**Ex) Full Search**
(10000 Evaluation)

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About 33 hours

Case Study: MILES Model - Result

Proposed algorithm is much faster than FS
SS-10 (393 times), SS-20 (220 times), SS-30 (153 times)

Parameter Setting: Alpha 0.01, 1000 replications, StepSize(SS) 10 ~ 30

Conclusion & Future Works

- Conclusion
  - Propose simulation based optimization algorithm for effectiveness analysis model
  - Using the hypothesis test, classify input scenarios
  - Increase n for input scenarios which has a probability to be the optimal solution and decrease n for the others
    ➔ That makes the algorithm use a limited budget efficiently.
  - Show enormous improvement of performance (Speed and Accuracy)

- Future Works
  - Apply the proposed algorithm to general stochastic model (noise model)
  - Expand the algorithm to reverse simulation framework