Effectiveness Analysis Model

Effectiveness Analysis Model \( f(X) \)

MOP
Speed, Operation Time, ...

MOE
Success(1)/Fail(0), Survive(1)/Destroy(0), ...

Find \( p_i \) (Measure of Effectiveness, MOE) from given \( X_i \) (Measure of Performance, MOP)

\[ f(X_i) = Y_i \sim Bernoulli(p_i) \]

\[ p_i = \lim_{n \to \infty} \frac{\sum_{k=1}^{n} Y_i}{n} \]
Simulation Based Optimization (1/2)

- For Effectiveness Analysis Model
  - Find input scenarios that maximizes(or minimizes) $p_i$

\[
X_o = \arg \max_{X \in \Theta} p_i = \arg \max_{X \in \Theta} \left[ \lim_{n \to \infty} \sum_{k=1}^{n} f(X_i)/n \right]
\]

- In real world, infinite evaluations is impossible.
  - Get point estimate $\hat{p}_i$ by limited $n$-th evaluation

\[
\hat{p}_i = p_i + N(0, p_i(1 - p_i)/n) = p_i + \text{noise}
\]

How can we find input scenarios that maximizes(or minimizes) $p_i$ with reducing an effect of the noise in limited evaluations?
Simulation Based Optimization (2/2)

• To minimize an effect of the *noise*
  – Increase *n* (number of evaluations)
    • Increasing *n* for all input scenarios is inefficient
  – Increase *n* for input scenarios which has a probability to be the optimal solution and decrease *n* for the others

![Graphs showing allocation of evaluations](image)

More likely to be the optimal solution ➔ Increase *n*

The others ➔ Decrease *n*

Proposed Algorithm
Proposed Algorithm (1/2)

1. **Candidate Set** \( (X_1, X_2, \ldots, X_k) \)

2. **Parameter:** StepSize, Alpha(\(\alpha\)), Budget

3. **StepSize Evaluations** (For All Candidates)
   - \( u_0 \) is a \((1 - \alpha)\) lower-bound of \(\text{max}(\hat{p}_i)\)
   - Calculate \( p\)-value set \((pV_1, pV_2, \ldots, pV_k)\)
   - Reduce Candidate Set (if \( pV_i < \text{Alpha} \), Delete it)

4. **Decide \( u_0 \)**
   - \( u_0 \) is a \((1 - \alpha)\) upper-bound of \(\text{min}(\hat{p}_i)\)
   - Calculate \( p\)-value of below hypothesis test\(^*\)

5. **When you find minimum**
   - No Estimated Points, \( u_0 \)
   - New Candidate Set \((X_1, X_2, \ldots, X_{k'})\)

6. **Remaining Budget?**
   - Yes
   - Decide Optimal Solution (Analysis of Independent Sample)\(^{[1]}\)
   - Optimal Solution Set
   - Yes
   - No

7. **Optimal Solution Set**

\(^*\)if \( \hat{p}_i = 0 \) or 1 then \( pV_i = 0 \) or 1

Proposed Algorithm (2/2)

- Hypothesis Test\[1\]
  - $H_0: \hat{p}_i \geq u_0, H_A: \hat{p}_i < u_0$
  - $u_0 = \max(\hat{p}_i) - t_{\alpha,n-1} \times \sqrt{\max(\hat{p}_i)(1 - \max(\hat{p}_i))/n}$

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Case Study: MILES Model (1/2)

- **Multiple Integrated Laser Engagement System (MILES)**
  - Find input scenarios to maximize effectiveness (hit rate)
  - Input scenario = [Beam Width, Sensing Degree, # of Sensor, Location of Sensor, Target]

\[
X \xrightarrow{\text{MILES Effectiveness Analysis Model}[1]} Y(\text{hit rate}) \sim \text{Bernoulli}(p_i)
\]

*Result from 100000 Evaluations for each $X$

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Case Study: MILES Model (2/2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Width (cm)</td>
<td>{ 5, 10, ..., 100 } (20)</td>
</tr>
<tr>
<td>Sensing Degree (°)</td>
<td>{ 5, 10, ..., 90 } (18)</td>
</tr>
<tr>
<td># and location of Sensor</td>
<td>{ P_1, P_2, ..., P_{63} } (63)</td>
</tr>
<tr>
<td>Target</td>
<td>{ T_1 } (1)</td>
</tr>
</tbody>
</table>

# of Input \(20 \times 18 \times 63 \times 1 = 22680\)
Case Study: MILES Model - Result

Proposed algorithm is much faster than FS SS-10 (393 times), SS-20 (220 times), SS-30 (153 times)

Parameter Setting: Alpha 0.01, 1000 replications, StepSize(SS) 10 ~ 30
Conclusion & Future Works

• Conclusion
  – Propose simulation based optimization algorithm for effectiveness analysis model
  – Using the hypothesis test, classify input scenarios
  – Increase $n$ for input scenarios which has a probability to be the optimal solution and decrease $n$ for the others
    ➔ That makes the algorithm use a limited budget efficiently.
  – Show enormous improvement of performance (Speed and Accuracy)

• Future Works
  – Apply the proposed algorithm to general stochastic model (noise model)
  – Expand the algorithm to reverse simulation framework

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